Wealth, Income, Earnings and the Statistical Mechanics of Flow Systems

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0.0 Abstract

This paper looks at empirical data from economics regarding wealth, earnings and income, alongside a flow model for an economy based on the general Lotka-Volterra models of Levy & Solomon.

The data and modelling suggest that a simple economic system might provide a tractable model for giving an exact statistical mechanical solution for an 'out of equilibrium' flow model. This might also include an exact mathematical definition of a 'dissipative structure' derived from maximum entropy considerations.

This paper is primarily a qualitative discussion of how such a mathematical proof might be achieved.

Contents

0.2 Introduction

This paper is a condensed extract from the full paper 'Why Money Trickles Up' which is available at econodynamics.org. The paper below is confined to a brief discussion of empirical data, mathematical background and modelling outputs. Background explanation and justification of assumptions has been kept to a minimum. The numbering in this paper follows that of 'Why Money Trickles Up', this may mean that numbering of equations may occasionally be out of order.

'The paper 'Why Money Trickles Up' is a wide-ranging paper that looks at the application of statistical mechanics and dynamics across economics and finance. It was found that a simple modelling approach, based on Lotka-Volterra and General Lotka-Volterra models had widespread application, giving solutions to wealth and income distributions, company size distributions, and also commodity and business cycles.

Unlike the exchange models widely used in econophysics, the GLV models used to generate the wealth and income distributions are 'flow' models, these are 'out of equilibrium' thermodynamic models, though they do come to dynamic equilibrium. It is the belief of the author that these flow models are maximum entropy flow models, analogous to the models described by Paltridge, Lorenz, Dewar and others primarily in the field of planetary ecology.

Although the models in 'Why Money Trickles Up' explained many aspects of wealth and income distribution, the source of earnings distributions; the distributions of wages and salaries, remains unexplained. It is the belief of the author that these distributions, which give very good fits to Maxwell-Boltzmann distributions, may allow the simple mathematical description of 'flow' systems by modelling them as a special variant of exchange models.

Section 1.1 of this paper briefly reviews the background empirical information on wealth and income distributions. Section 1.2 briefly summarises the Lotka-Volterra and General Lotka-Volterra models, while section 1.3 briefly reviews the outputs from the wealth and income modelling of 'Why Money Trickles Up'. Section 7.3 reviews the history of 'Maximum Entropy Production' and section 7.4 discusses the proposed approaches for modelling the statistical mechanics of 'out of equilibrium' thermodynamic systems.

1.1 Wealth & Income Data – Empirical Information

Vilfredo Pareto first showed in 1896 that income distributions followed the power law distribution that now bears his name [Pareto 1896]. Despite the differences between the economies studied, Pareto discovered that the income of wealthy individuals varied as a power law in all cases. Research to the present day has confirmed this.

Typical graphs of income distribution are shown below. This is data for 2002 from the UK, and is an unusually good data set [ONS 2003].

Figure 1.1.1 here

Figure 1.1.1 above shows the probability density function. On these scales the log-normal appears to give a very good fit to the data.

Figure 1.1.2 here

Figure 1.1.2 above shows the same data, on a log-linear scale. The log-normal fit cannot describe the income of high-earners, above £900.

Figure 1.1.3 here

Figure 1.1.3 above shows the same data as a cumulative density function. The straight line Pareto power tail section is on the right-hand side.

The Pareto distribution actually only applies to the top 10%-20% of earners, though it might include 50% of income. The other 80%-90% of middle class and poorer people are accounted for by a different 'body' of the distribution. This is typically modelled by a log-normal distribution. The author has suggested that a Maxwell-Boltzmann distribution also provides a good fit to the main body of the income data that is equal to that of the log-normal distribution [Willis & Mimkes 2005].

The reasons for the split between the income earned by the top 10% and the main body 90% has been studied in more detail by Clementi and Gallegati [Clementi & Gallegati 2005a]. This shows strong economic regularities in the data. This suggests that the income gained by individuals in the power tail comes primarily from income gained from capital, while the income for the 90% of people in the main body of the distribution is primarily derived from wages. This view is supported by a data set form the US Department of Labor; Bureau of Statistics shown in figures 1.1.4 and 1.1.5.

Figure 1.1.4 here

Figure 1.1.5 here

While this data has main bodies identical to the UK data, there is no sign whatsoever of the 'power tail', it is the belief of the author that the methodology for this US survey restricted the data to 'earned' or 'waged' income, as the interest in the project was in looking at pay in services versus manufacturing industry. It is believed income from assets and investments was not included as this would have been irrelevant to the investigation.

This US data set has been included for a further reason, like the UK data of figure 1.1.1 there is a very clear offset from zero along the income axis.

This is important, as both the log-normal and Maxwell-Boltzmann distributions normally start at the origin of the axis. While it is straightforward enough to put an offset in, this is not normally necessary when looking at natural phenomena.

In recent years, the study of income distributions has gone through a small renaissance in the field of 'econophysics', see [Gabaix 2009, Chatterjee et al 2007, Chatterjee & Chakrabarti 2007, Sinha 2005 Bouchaud & Mezard 2000, Dragulescu & Yakovenko 2001, Nirei & Souma 2007, Souma 2001, Slanina 2004]. The majority of these papers follow similar approaches; inherited either from the work of Gibrat, or from gas models in physics. Almost all the above models deal with basic exchange processes, with some sort of asymmetry introduced to produce a power tail.

All these models follow a traditional approach that assumes a static thermodynamic equilibrium.

It is important to note the difference between income and wealth. Trivially income is the time derivative of wealth. Without exception all the exchange models by all the various authors above, including those of Levy and Solomon, are wealth exchange models. I have not yet seen a model where income is an output. Despite this, the output distributions from these wealth models are often judged to be successful when they map well onto data derived from income studies.

An alternative approach to stochastic modelling has been taken by Moshe Levy, Sorin Solomon, and others [Levy & Solomon 1996].

They have produced work based on the 'General Lotka-Volterra' model.

A brief discussion of the origin and mathematics of GLV distributions is given below in section 1.2.

Figure 1.1.6 here

Figure 1.1.7 here

Figures 1.1.6 and 1.1.7 above show plots of UK income data against the GLV. The GLV is preferred for a number of reasons; these include the following:

The GLV has both a power tail and a 'log-normal'-like main body. Other distributions investigated only model one or other of these.

The GLV has a 'natural' offset from zero. For the GLV the rise from zero probability starts at a non-zero x value.

Thirdly the detailed fit of the GLV appears to be equivalent or better than the log-normal distribution, see Fig 1.1.8.

Figure 1.1.8	Reduced Chi Squared	
	Full Data Set	Reduced Data Set
Boltzmann Fit	3.27	1.94
Log Normal Fit	2.12	3.02
GLV Fit	1.21	1.83

Remarkably, the figures in the second column show the GLV gives a marginally better fit even when the power law data is excluded.

However the main reason for using the GLV is that it allows an effective, intuitive and simple economic formulation. This is the main reason for preferring the GLV distribution, and is discussed in depth in the paper 'Why Money Trickles Up' (henceforth YMTU) [Willis 2011b]. In YMTU it is demonstrated that the (multiplicative) GLV can very effectively model wealth distributions. This then also gives an apparent GLV for total income.

It is the belief of the author that the earnings distributions from the US are in fact the product of a dynamic equilibrium process that produces an 'additive GLV' distribution, in contrast to the 'multiplicative GLV' distributions that are produced for wealth.

Background on the formation of power laws, log-normal laws and related processes, is given in three very good papers by Newman [Newman 2005], Mitzenmacher [Mitzenmacher 2004] and Simkin & Roychowdhury [Simkin & Roychowdhury 2006].

One basic point from the papers is that there are many different ways of producing power law distributions, but the majority fall into three main classes.

The first class gives a power law distribution as a function of two exponential distributions; of two growth processes.

The second class gives power law distributions as an outcome of multiplicative models. This is the route that Levy and Solomon have followed in their work, and forms the basis for the GLV distribution discussed in detail in the next section.

The third class for producing power laws uses concepts of 'self-organised criticality' or 'SOC'.

1.2 Lotka-Volterra and General Lotka-Volterra Systems

1.2.1 Lotka-Volterra systems

Lotka-Volterra systems [Lotka 1925, Volterra 1926] are used to describe the dynamics of populations in ecological systems. A basic Lotka-Volterra system consists of a population of prey (say rabbits) whose size is given by x, and a population of predators (say foxes) given by y. When no predators are present this means that the population of the rabbits is governed by:

$$\frac{dx}{dt} = ax \qquad (1.2.1a)$$

where a is the population growth rate.

This would give exponential growth in the population of the rabbits. In the absence of any rabbits to eat, it is assumed that there is a natural death rate of the foxes:

$$\frac{dy}{dt} = -cx \qquad (1.2.1\,b)$$

where c is the population die-off rate. This would give an exponential fall in the fox population.

When the foxes encounter the rabbits, two further effects are introduced, firstly the rate at which rabbits are killed is proportional to the number of rabbits and the number of foxes (ie the chance of foxes encountering rabbits), so:

$$\frac{dx}{dt} = -\alpha x y \qquad (1.2.1\,c)$$

where α is a constant, and the –ve sign indicates that such encounters are not good for the rabbits. However these interactions are good for the foxes, giving:

$$\frac{dy}{dt} = \gamma x y \qquad (1.2.1d)$$

Where γ is again a fixed constant.

Taken together, the results above give a pair of differential equations:

$$\begin{aligned}\frac{dx}{dt} &= ax - \alpha x y \\ &= x(a - \alpha y) \qquad (1.2.1\,e)\end{aligned}$$

for the rabbits, and:

$$\begin{aligned}\frac{dy}{dt} &= \gamma x y - cy \\ &= y(\gamma x - c) \\ &= y(-c + \gamma x) \qquad (1.2.1f)\end{aligned}$$

for the foxes.

1.2.2 General Lotka-Volterra (GLV) systems

As the name implies, the General Lotka-Volterra system (GLV) is a generalisation of the Lotka-Volterra model to a system with multiple predators and prey. This can be represented as:

$$\frac{dx_i}{dt} = x_i r_i + \sum_{j=1}^{N} a_{i,j} x_i x_j \qquad (1.2.2a)$$

$$= x_i (r_i + \sum_{j=1}^{N} a_{i,j} x_j) \qquad (1.2.2b)$$

here, dx_i/dt is the overall rate of change for the i-th particular species, out of a total of N species. This is made up of two terms.

The first term is the natural growth (or death) rate, r_i, for the species, where x_i is the population of species i. This rate r_i is equivalent to the growth rate 'a' in equation (1.2.1e) or the death rate '-c' in equation (1.2.1f).

The second term gives the sum of all the interactions with the j number of other species. Here $a_{i,j}$ is the interaction rate defining the relationship between species i and j.

$a_{i,j}$ is negative if species j is a predator feeding on species i, positive if species i is a predator feeding on species j, or can be of either sign for a heterotroph. $a_{i,j}$ is equivalent to the α of equation (1.2.1e) or the γ of equation (1.2.1f).

Equations (1.2.2a) and (1.2.2b) are generalisations of equations (1.2.1e) and (1.2.1f) for many interacting species.

For each species in the system, potentially N-1 interaction rates $a_{i,j}$ are needed, while N! separate differential equations are needed to describe the whole system, which is problematic.

Fortunately in many systems it is possible to make simplifying assumptions. As an example Solomon [Solomon 2000] proposes the following difference equation as a possible explanation for the power law distribution of city population sizes. This equation describes changes in the distribution in terms of discrete time-steps from time t to time t+1:

$$w_{i,t+1} = \lambda_t w_{i,t} + a_t \bar{w}_t - c_t \bar{w}_t w_{i,t} \qquad (1.2.2c)$$

The terms on the right hand side, in say the year 2003, the year t, add up to give the population w of city i in the year 2004 on the left hand side, which is at time t+1.

In equation (1.2.2c), λ is the natural growth rate of the population w of city i, but is assumed that λ is the same for each city. a_t is the arrival rate of population from other cities, which is multiplied by the average population $\bar{w}$ of all the cities. The final term gives the rate of population leaving each city, which is due to the probability c_t of an individual meeting a partner from another city. This is given by multiplying the average population $\bar{w}$ with the population of city i.

Leaving aside the detail of the model, important generalisations have been made to produce a more tractable model.

In this case λ, a and c are universal rates, applicable to all members of the system.

λ and a both give 'positive autocatalytic' (positive feedback) terms which increase the population w of each city. While the negative value of c ensures that the population of each city has an element of decrease.

In the absence of the negative feedback term, the populations of the cities can increase indefinitely to infinity without reaching a stable solution.

In the absence of the positive autocatalytic growth of the λ in the first term on right hand side, the second and third terms will cause all of the population to end up in a single city.

Normally one or more variables are assumed to be stochastic; that is they can vary randomly. In Solomon's example above, all three of λ, a and c are assumed to be stochastic. This stochasticity need not be large; it can be small fluctuations around a long-term mean, but it ensures that a locally stable solution is not reached, and that the system evolves into a single long term equilibrium solution.

By moving from the Lotka-Volterra model to a general Lotka-Volterra model, with numerous agents, Levy & Solomon have moved from a chaotic/dynamic problem to a statistical mechanical problem. In their mathematical analysis of the GLV, Levy and Solomon show that the entropy of multiple Boltzmann distributions gives the power law tails found in the GLV distribution [Levy & Solomon 1996].

Levy and Solomon show that the above system can give a stable resultant probability distribution function of populations over the various cities of the form:

$$P(w) = (e^{-(\alpha-1)/w})/(w^{(1+\alpha)}) \qquad (1.2.2d)$$

Which is the general form of the GLV distribution. Or more specifically:

$$P(w) = K(e^{-(\alpha-1)/(w/L)})/((w/L)^{(1+\alpha)}) \qquad (1.2.2e)$$

1.3 Wealth & Income Modelling

The author carried out a modelling exercise that was successful in modelling wealth and income distributions. This is discussed in full in the paper 'Why Money Trickles Up' [Willis 2011b]. This was based on the model shown in figure 1.3.5.

Figure 1.3.5 here

The income stream Y has been split into two components; e is the earnings; the income earned from employment as wages and salaries, in return for the labour supplied.

$$Y = e + \pi$$

π is the 'profit' and represents the payments made by firms to the owners of capital, this can be in the form of dividends on shares, coupons on bonds, interest on loans, rent on land or other property, etc.

All of this real capital is assumed to be owned by households, in the form of paper assets, W, representing claims on the real assets in the form of stocks or shares. More complex assets are ignored, as are personal assets such as housing, cars, jewellery, etc. It is assumed that all the real wealth K is owned in the form of shares (stocks) in the various firms.

This paper wealth is represented as W in total, or w_i for each of i individuals.

For the income models it was further assumed that the paper wealth of the households accurately represents the actual physical capital owned by the companies, so the total real capital invested in the firms is equal to the total value of financial assets held by individuals.

In this model it was assumed that there was a steady state, so the totals of W and K are both constant. This means that the model has no growth, and simply continues at a steady equilibrium of production and consumption. So:

$$\text{total } C = \text{total } Y = \text{total } My$$

Note that although the totals of C and Y are the same, they may not be the same for individuals. Some individuals may consume less than they earn, or vice versa.

Other important base assumptions of the model were:

- The economy is isolated; there are no imports or exports.
- There is no government sector, so no taxation or welfare payments, government spending, etc.
- There is no unemployment; all individuals are employed, with a given wage, either from a uniform distribution or a normal distribution depending on the model.
- Labour and capital are assumed to be complementary inputs and are not interchangeable at least in the short term.
- The role of money is ignored.
- There is no debt included in the income models.

The paper wealth W will be split between N individuals, although the total capital and wealth is fixed, individual wealth is allowed to vary, so:

$$\sum w_{i,t} = \sum w_{i,t+1} = W = K = \text{constant} \qquad (1.3e)$$

Where w_i is the wealth of individual i.

Looking at a single individual in the box on the right of figure 1.3.5, in one time unit, from t to t+1, the change in wealth is given by the following equation:

$$w_{i,t+1} = w_{i,t} + e_{i,t} + \pi_{i,t} - C_{i,t} \qquad (1.3h)$$

In a single iteration, the paper wealth w of an individual i increases by the wages earned e plus the profits received π. The individual's paper wealth also reduces by the amount spent on consumption C.

Although the totals of My = Y = C some individuals can consume less than their income, and so accumulate more wealth W, others can consume more than their income and so reduce their total W.

Looking at the third and fourth terms on the right hand side of (1.3h) in the first model:

Firstly π, the income from returns. It is assumed that the economy consists of various companies all with identical risk ratings, all giving a uniform constant return; r on the investments owned, as paper assets, by the various individuals. Here r represents profits, dividends, rents, interest payments, etc. To prevent confusion with other variables, r will normally be referred to as the profit rate.

This gives:

$$\pi_{i,t} = w_{i,t} r \qquad (1.3j) \quad \text{for each of the i agents.}$$

For the final term consumption; C is assumed to be a simple linear, stochastic, function of wealth. So:

$$C_{i,t} = w_{i,t} \Omega \qquad (1.3n)$$

Taken together and substituting into (1.3h) this gives the difference equation for each agent as follows:

$$w_{i,t+1} = w_{i,t} + e_{i,t} + w_{i,t} r - w_{i,t} \Omega \qquad (1.3o)$$

Equation (1.3o) is the base equation for a single agent in all the income models. It is worth noting how simple this equation is. Here w is the only variable. e, r and Ω are all constants; e and r are absolute constants Ω is constant over the long term, but is stochastic in the short term.

The second term on the RHS, the earned income e, provides a constant input that prevents individual values of wealth collapsing to zero, it also gives the offset from zero in the shape of the GLV distribution. Note that this is additive, where in the models of Levy & Solomon in section 1.2 above this term was multiplicative. In the modelling of YMTU the earnings distribution $e_{i,t}$ was defined exogenously as either a uniform or normal distribution.

The third term on the RHS is a multiplicative term and gives a positive feedback loop. The fourth term is also multiplicative and gives negative feedback.

There is an important subtlety in the model above. In the original textbook economic model the total income and consumption are made equal by definition. In the models in this paper, income is fixed, but consumption varies with wealth. The negative feedback of the final consumption term ensures that total wealth varies automatically to a point where consumption adjusts so that it becomes equal to the income.

This automatically brings the model into equilibrium. If income is greater than consumption, then wealth, and so consumption, will increase until C=Y.

If income is less than consumption, the consumption will decrease wealth, and so consumption, until again, C=Y.

Most income models produced in econophysics are exchange models. In these exchange models total wealth is fixed exactly and wealth is simply exchanged during the running of the model.

The GLV model of YMTU discussed above is a flow model, it is an 'out of equilibrium' model in terms of traditional thermodynamics. Total wealth is not fixed in the model. Wealth is continuously created and destroyed. However, because of the self-regulating consumption term, total wealth does come to a constant value. The system does come to a dynamic equilibrium, in the sense that all the parameters of the system come to constant values.

In all the income models studied, the total income Y per time unit was fixed, and unless otherwise specified, the earned income was fixed equal to the returns income. So:

$$Y = \sum e_i + \sum \pi_i = \text{constant}, \quad \text{always} \qquad (1.3p) \quad \text{and}$$

$$\sum e_i = \sum \pi_i = \frac{Y}{2} \qquad \text{usually} \qquad (1.3q)$$

So unless otherwise specified, the total returns to labour are equal to the total returns to capital. This accords with the real world where, the share of labour earnings out of total income can vary typically between 0.75 and 0.5. This is known as Bowley’s Law, and represents as close to a constant as has ever been found in economics, figure 1.3.8 below gives an example for the USA. In developing economies, with pools of reserve subsistence labour, values can vary more substantially. Young gives a good discussion of the national income shares in the US, noting that the overall share is constant even though sector shares show long-term changes [Young 2010]. Gollin gives a very thorough survey of income shares in more than forty countries [Gollin 2002].

Figure 1.3.8 here

[St Louis Fed 2004]

Because of the importance of Bowley's law, it is useful to define some ratios:

$$\text{Profit rate} \quad r = \frac{\sum \pi}{W} \qquad (1.3r)$$

Where profit can refer to any income from paper assets such as dividends, rent, coupons on bonds, interest, etc.

$$\text{Income rate} \quad \Gamma = \frac{Y}{W} \qquad (1.3s)$$

which is the total earnings over the total capital. Here total earnings is all the income from wages and all the income from financial assets added together.

$$\text{Bowley ratio} \quad \beta = \frac{\sum e}{Y} \qquad (1.3t)$$

$$\text{Profit ratio} \quad \rho = \frac{\sum \pi}{Y} \qquad (1.3u)$$

These two define the wages and profit respectively as proportions of the total income. Following from the above, the following are trivial:

$$\beta + \rho = 1 \qquad (1.3v)$$

$$\text{Profit ratio} \quad \rho = \frac{r}{\Gamma} \qquad (1.3w)$$

$$\text{Bowley ratio} \quad \rho = 1 - \frac{r}{\Gamma} \qquad (1.3x)$$

We also define:

$$\text{Consumption rate} \quad \Omega = \frac{C}{W} \qquad (4.5c)$$

and given that:

$$\text{Consumption} = \text{Income}$$

$$C = Y \qquad (4.5b)$$

Then clearly:

$$\Omega = \Gamma \qquad (4.5n)$$

This then gives alternative definition of the profit ratio and Bowley ratio that emerged from the modelling of YMTU:

$$\rho = \frac{r}{\Omega} \qquad (4.5h)$$

$$\begin{aligned} \beta &= \text{Bowley ratio} \\ &= 1 - \frac{r}{\Omega} \qquad (4.5k) \end{aligned}$$

Both the above are derived trivially by substituting from (4.5n) into (1.3w) and (1.3x). In fact (4.5h) and (4.5k) give the fundamental definitions of ρ and ß. For a full explanation of this see the full paper 'Why Money Trickles Up' or the shorter paper 'The Bowley Ratio' [Willis 2011a, 2011b].

Using a modelling approach based on equation 1.3o it was possible to generate full explanations of both wealth and income distributions, examples are shown in figures 1.4.4.1 to 1.4.4.4.

Figure 1.4.4.1 here

Figure 1.4.4.2 here

Figure 1.4.4.3 here

Figure 1.4.4.4 here

It can be seen from the figures above that the modelling gave full fits for the GLV to both wealth and income. For a full discussion of the results, see the full paper 'Why Money Trickles Up'.

These models explain the distributions for wealth fully, they also explain the distributions of income as a sum of investment income and and an externally defined, exogenous, distribution of earned income. The models do not explain the origin of this distribution of earned income, that is the main point of the discussions further in this paper.

An investigation was carried out into the effect of the profit ratio and Bowley ratio, along with other modelling inputs, on the value of α, the key parameter of the GLV in equation (1.2.2). It can be seen in figure 1.6.5 that there appears to be a strong relationship between ρ and α.

Figure 1.6.5 here

From the modelling it was possible to extract the following relationships for α, here v is the variance of the distribution of Ω.

$$\alpha = \frac{1.36(1 - \rho)}{v^{1.15}} \qquad (1.6d) \qquad \text{or:}$$

$$\alpha = \frac{1.36\beta}{v^{1.15}} \qquad (1.6e)$$

Equations (1.6d) and (1.6e) are deceptively simple and appealing, and their meaning is discussed below in more detail.

Before this is done, it is worth stressing some caveats.

Firstly the two equations (1.6d) and (1.6e) have been extracted empirically from a model. They have not been derived mathematically. Neither have they been extracted from real data. Although it is the belief of the author that the equations are important and are sound reflections of economic reality, this remains solely a belief until either the equations are derived exactly or supporting evidence is found from actual economic data; or, ideally, both.

Secondly the nature of the two variables β and v are different. The Bowley ratio is well known in economics and is an easily observed variable in national accounts. In contrast v is the variance in an assumed underlying distribution of consumption saving propensity. In real economics the shape of such a distribution is controversial and is certainly not settled. Indeed the discussions of this paper are an attempt to resolve this issue.

Although equation (1.6e) is simpler, equation (1.6d) is the key equation here. Indeed the more diligent readers may have noted the strong resemblance of equation (1.6d) with the exponent produced from equation (45) in Newman [Newman 2005], which gives a general formula for α as:

$$\alpha = 1 - a/b \qquad (1.6f)$$

Where a and b are two different exponential growth rate constants.

This is of course exactly what we saw in equation (1.3w) where ρ is the ratio of two different growth constants, r and Γ.

$$\text{Profit ratio} \qquad \rho = \frac{r}{\Gamma} \qquad (1.3w)$$

The value of ρ is simply the growth rate that capitalists get on capital, divided by the growth rate that everybody (capitalists and workers) gets on capital.

It is the combination of these two growth rates that creates and defines the power law tail of the wealth and income distributions. This is the first, and simplest class of ways to generate power laws discussed in Newman [Newman 2005].

And a curious thing has happened here. There are many different ways to produce power laws, but most of them fall into three more fundamental classes; double exponential growth, branching/multiplicative models, and self-organised criticality.

The models in YMTU were firmly built on the second group. The GLV of Levy and Solomon is a multiplicative model built along the tradition of random branching models that go back to Champernowne in economics and ultimately to Yule and Simon [Simkin & Roychowdhury 2006].

Despite these origins we have ended up with a model that is firmly in the first class of power law production, the double exponential model.

It is the belief of the author that this is because the first two classes are inherently analogous, and are simply different ways of looking at similar systems.

Much more tentatively, it is also the belief of the author that both the first two classes are incomplete descriptions of equilibrium states, and further input is need for most real systems to bring them to the states described by the third class; that of self organised criticality (SOC).

Going back to the wealth and income distributions; equation (1.6d) can define many different possible outcomes for α. Even with a fixed Bowley ratio of say 0.7, it is possible to have many different values for α depending, in this case, on the value of v.

It is worth noticing that there is a mismatch between the values for α given by the models and economic reality. The models give values of α of 4 and upwards for both wealth and income. In real economies the value of alpha can vary in extreme cases can between 1 and 8, but is typically close to a value of 2 see for example Ferrero [Ferrero 2010].

It is the belief of the author that in a dynamic equilibrium, the value of α naturally tends to move to a minimum absolute value, in this case by maximising v to the point where the model reaches the edge of instability. At this point, with the minimum possible value of α (for any given value of ρ or β) there is the most extreme possible wealth/income distribution, which, it is the belief of the author is a maximum entropy, or more exactly a maximum entropy production, equilibrium. This belief; that self-organised criticality is an equilibrium produced by maximum entropy production, is discussed in more detail in section 7.3 below.

It is the suspicion of the author that the unrealistic distribution for Ω used in the modelling approach above results in a point of SOC, that is artificially higher than that in real economies. Indeed, it is a suspicion that movement towards SOC may of itself help to define underlying distributions of earnings and consumption. This is returned to in section 7.4.

7.3 Maximum Entropy Production

The models of 'Why Money Trickles Up' can not be described by standard models from the physics of thermal equilibrium.

The models in the paper consist of sources of wealth generation in companies and sinks of consumption at households, with a continuous flow from one to the other. In this they resemble models that have continuous flows of heat in and out of the system and that have different temperatures in different parts of the model.

These are 'out of equilibrium thermodynamics systems', or simply 'non-equilibrium systems'; though it is the belief of the author that this nomenclature may need to be revisited.

Traditionally, such systems have been very difficult to describe mathematically, however recent work by Lorenz, Paltridge, Ackland & Gallagher, and others in the field of planetary ecology and, also that of Dewar, Levy, Solomon and others in the field of theoretical physics appear to have changed things substantially.

In early models Paltridge and Lorenz discovered that the earth appeared to act in a 'deliberate' manner, by adjusting the temperatures across the globe, to a give a maximum possible rate of entropy production. Although it is early days, this principle of 'Maximum Entropy Production' or 'MEP' appears to be widely applicable, and also appears to make many previously insoluble systems much more tractable.

Analysis by other authors suggests that the same Maximum Entropy Production principle is true for the re-radiation of heat from Mars and Titan. It also appears that the use of MEP may be applicable to many other systems such as convection in the earth's mantle, and turbulent systems. Ozawa et al give an excellent review of the history and uses of MEP, while the book edited by Kleidon & Lorenz gives much more detail [Ozawa et al 2003, Kleidon & Lorenz 2005].

In Paltridge's model, earth becomes what is known as a 'dissipative structure'. Dissipative structures include things such as planets, and life forms. Dissipative structures are counter-intuitive from a normal equilibrium thermodynamic point of view. Dissipative structures are highly concentrated, highly organised, and so have very low entropy. From the point of view of ordinary equilibrium thermodynamics, they shouldn't exist.

However from an MEP point of view, dissipative structures do make sense. The existence of the low entropy structures facilitates more rapid entropy flow through the system as a whole.

A classic example is that of Bénard convection cells. The convection cells are low entropy structures, but they allow much more rapid transfer of heat from the bottom of the cells to the top.

Similarly, the earth's atmosphere operates as a dissipative structure moving hot equatorial air to the poles. The circulation of the oceans carries out exactly the same functions.

Interestingly, it also appears that the existence of plants changes the earths albedo in ways that also maximises entropy production. Animals, then appear as efficient redistributors and processors of vegetable matter.

When looked at in this manner almost everything on planet earth becomes a dissipative structure. This includes of course human society, and indeed, human economic systems.

In parallel with the above work in the field of ecology; Levy, Solomon and various co-workers have carried out pioneering theoretical work looking at the dynamics of the Generalised Lotka-Volterra distribution and how it works mathematically.

In their mathematical analysis of the GLV, Levy and Solomon show that the entropy of multiple Boltzmann distributions gives the power law tails found in the GLV distribution [Levy & Solomon 1996].

In contrast the Maxwell-Boltzmann distribution of a normal thermodynamic equilibrium comes from an additive process. This is a direct conservation law, in such a system the addition and subtraction are direct and total energy is conserved absolutely. This results in a distribution with an exponential tail.

The GLV comes from a multiplicative process. And multiplicative process cannot be directly conservative. The GLV process does however remain conservative in total, at least in the long term; the process of this conservation is discussed further below.

Because of its multiplicative nature, the output of the GLV includes a power law tail.

This can be seen as analogous to the central limit theorem.

Under the CLT an additive process gives a normal distribution, a multiplicative process gives a log-normal distribution, with an exponential tail.

Under an additive, maximum entropy process, the output is a Maxwell-Boltzmann distribution, with an exponential tail. Under a multiplicative, maximum entropy production process, the product is a GLV distribution, with a power tail.

'One sees therefore that a power law is as natural and robust for a stochastic multiplicative process as the Boltzmann law is for an equilibrium statistical mechanics system. Far from being an exception and requiring fine tuning or sophisticated self-organising mechanisms, this is the default.' [Levy & Solomon 1996]

As such, the GLV distribution might better be considered to be a 'log-Maxwell-Boltzmann' distribution.

Within the fields of ecology, these ideas have been taken forward in some very interesting work by Ackland & Gallagher [Ackland & Gallagher 2004] on the modelling of ecosystems. This modelling shows that, by using simple GLV models, and some very basic assumptions it is possible to produce full food webs with all the complexity of a real ecosystem. This model allows and includes for constant evolution and transformations of predators and prey within the system. Despite this the overall parameters of the food web become highly stable in things such as numbers of predators, prey, varieties of species, etc.

It is particularly interesting that a large array of different species, different types of dissipative structures, appears so as to maximise the total biomass flow.

"We monitored this during our simulations and found a remarkable result—the total flow of resource (and hence total biomass) increases with time reaching a plateau after many thousands of steps—the steady-state linkstrength ensemble distribution appears to be the one which maximizes the use of resource. This type of optimisation is consistent with what has been observed in other ecological models. If the model is recast in terms of flow and dissipation, the maximization principle is equivalent to maximum entropy production: the mathematical equivalent of "entropy production" is just the total death rate, and hence the flow out." [Ackland & Gallagher 2004]

It is the belief of the author that the economies of the world are acting in exactly the same manner. An economy is an MEP dissipative structure, and when it is at equilibrium it is maximising the rate of entropy production. It should be noted that this is not just an analogy. In entropy production terms, the human economic system is simply a complicated and interesting sub-section of the MEP function of the earth as a whole.

Returning to the mathematics, Dewar [Dewar 2005] has produced a seminal paper that derives maximum entropy production from the first principles of information theory and simple maximum entropy considerations.

This derivation of a Maximum Entropy Production (MEP) approach appears to be applicable to non-equilibrium systems in general.

Instead of looking at the counting of all possible statistical states, and finding the most probable, Dewar looks at the counting of all possible paths through a flow system, and finds that these can be counted using the same maximum entropy approach used by Boltzmann, Gibbs, etc.

Dewar does this by maximising the path information entropy, following the ideas of Shannon and Jaynes. This follows from Shannon's interpretations of information entropy and Jaynes generalisation of the maximum entropy approach as a general recipe for statistical inference.

I have not yet seen any theoretical work formally linking the work of Dewar to that of Levy & Solomon, however I am firmly convinced that they are isomorphous; that Levy & Solomon's mathematical derivations of the GLV should also be reproducible via working from Dewar's principals of path entropy.

It is my belief that Levy, Solomon and Dewar have produced some very important and very general principles. I believe that the max entropy production model, and GLV distributions will be found to give general and stable descriptions of many complex systems that have hitherto been seen as insoluble.

What Dewar, Levy and Solomon's systems consist of are three critical elements; a source, a sink, and some sort of self-limiting behaviour.

This model is potentially very powerful, as this simple model is typical of many complex systems. The sources and sinks are typically energy, but can also be population, or the wealth created in an economic system, or many other things.

The reason such systems are very common is because most other systems are inherently dull, at least in the longer term.

Without the source, the system quickly disappears.

Without the sink the system will quickly explode and disappear.

Without the self-balancing mechanism the system will either explode or disappear depending on the direction of the imbalance.

The self-balancing mechanism is the key to the long-term preservation of the process, and this reintroduces the conservation principle.

In a classical 'static' thermodynamic equilibrium conservation is absolute.

In a Dewar, Levy, Solomon type 'dynamic thermodynamic equilibrium', conservation is approximate and long term. Input and output can differ over the short term, but are brought back into balance automatically in the long term. Indeed such systems can wander backwards and forwards in a Lotka-Volterra type manner at a macroscopic level, while maintaining GLV type equilibrium at a microscopic level.

In economics the source is production, the sink is consumption. Going way back to section 1.2 and section 1.3 there was a discussion of the different ways of producing power laws. These different methods were combinations of two exponential processes, multiplicative process, and self-organised criticality (SOC). As discussed in section 1.3 it is the belief of the author that the first two processes, double exponentials and multiplicative processes are in fact different ways of describing the same process. In the GLV this becomes obvious if you look at the difference equation (1.3o), which can be seen as either a way of multiplying the variables (a multiplicative process) or a way of modelling two different growth rates (double exponential process). However a single GLV can have many different possible equilibria. Dewar ties this together, and shows that dynamic systems tend to a single point of maximum entropy production point, a single dynamic equilibrium, at the limit of stability, at the point of self organised criticality.

This appears to be typical of many systems, and may explain the fact that many power law distributions have values between two and three even though they arise from substantially different underlying models (see Newman table 1 for example [Newman 2005]).

Indeed Dewar points out that many very chaotic systems; systems close to 'self organised criticality' such as earthquakes, avalanches, forest fires and the archetypal sandpiles, can be characterised by slow steady underlying growth rates (eg tectonic plate movement for earthquakes, tree growth rate for forest fires). He also explains that such systems can be included in the Maximum Entropy Production modelling approach, even though such systems are traditionally characterised as being very far from equilibrium.

Financial markets, especially asset markets, also show many of the characteristics of such SOC systems with steady growth intermittently interrupted with dramatic crashes.

An example, for those that can remember them, is the traditional old-fashioned egg-timers. When well-built, these represented a very well behaved sandpile. In a high quality egg-timer, the sand is very fine, with equal sized smooth grains, the sand is dry and friction is very low. In such an egg-timer the sandpile has a near constant, flattish, inverted conical shape, and close observation shows that the avalanches are small but near-continuous. With a 'normal' sandpile the sand behaves much more erratically. With a little 'stickiness', caused by damp or a wide distribution of grain sizes, the pile can build up significantly into steeper and steeper hills as grains are added at the top. Eventually a dramatic collapse occurs which changes the steep hill into a much shallower one, then the process restarts.

In human managed forests this lesson has been learned, though at a cost. In the middle of the last century forest managers attempted to fight forest fires by removing undergrowth and ignition sources. This appeared to work in the short run, but eventually this simply led to much larger, and more devastating and dangerous fires. In recent decades foresters now often manage nature reserves by deliberately starting fires on a frequent basis. This results in a steady stream of much smaller fires.

For the reasons above, I believe that the nomenclature of such systems needs to be reviewed. In many cases I believe that many complex systems that are currently described as 'out of equilibrium' should be described as being in 'dynamic thermodynamic equilibrium' or 'MEP equilibrium'. This form of equilibrium is reached when the system has reached the point of maximum entropy production and continues indefinitely in that state.

7.4 The Statistical Mechanics of Flow Systems

In this section I would like to make some suggestions as to possible ways forward for a statistical analysis of the flow systems described by Levy, Solomon and Dewar.

I would like to do this by attempting to reduce these models to equivalent exchange models.

The modelling in YMTU does not use exchange models, primarily because they do not provide models that realistically capture the processes of real economic systems. For these reasons I have built the models in YMTU following the flow pattern of the GLV of Levy and Solomon. However for a core production of the statistical mechanics I believe appropriately designed exchange models may be useful proxies for flow models.

Very many exchange models have been produced by econophysicists, with many different underlying mechanisms. In a very perceptive paper; 'The Rich Are Different!: Pareto Law from asymmetric interactions in asset exchange models' [Sinha 2005] Sitabhra Sinha points out that these models share a very basic pattern. When these models have a symmetric pattern of exchange they produce a traditional Maxwell-Boltzmann distribution. When the exchange mechanism is made to be asymmetric, then a power law is produced. Indeed; in one case an asymmetric mechanism was deliberately introduced to assist the poor, but instead produced a power law tail; so giving the opposite result of that intended.

I believe it is a similar simple asymmetry that drives the multiplicative flow models of Levy, Solomon and Dewar.

If we go back to the base equation (1.3o) for a single agent in the economic models from section 1.3:

$$w_{i,t+1} = w_{i,t} + e_{i,t} + w_{i,t}r - w_{i,t}\Omega \qquad (1.3o)$$

I would firstly like to generalise this to the following:

$$w_{i,t+1} = w_{i,t} + e_{i,t} - \tau + w_{i,t}r - w_{i,t}\Omega \qquad (7.4a)$$

The term τ represents what economists normally call 'non-discretionary spending'. This is assumed (in my discussions) to be a base constant value that includes for basic housing, as well as minimum requirements for food, clothing, heating etc. All other spending is assumed to be discretionary, and proportional to wealth and so included in Ω.

If we now do a summation of equation (7.4a) across all individuals we get:

$$\sum w_{i,t+1} = \sum w_{i,t} + \sum e_{i,t} - \sum \tau + \sum w_{i,t}r - \sum w_{i,t}\Omega \qquad (7.4b)$$

Let us then assume that the dynamic flow model is at a dynamic equilibrium, ie that it is neither growing nor shrinking through time, though it is still flowing. At this equilibrium the total wealth is constant between times steps, so the term on the left hand side is equal in value to the first term on the right hand side. This gives:

$$0 = \sum e_{i,t} - \sum \tau + \sum w_{i,t}r - \sum w_{i,t}\Omega \qquad (7.4c)$$

The obvious way to balance this economic flow system is as an accounting identity as follows:

$$\sum e_{i,t} + \sum w_{i,t}r = \sum \tau + \sum w_{i,t}\Omega \qquad (7.4d)$$

This balances the total incomes on the left and the total consumption on the right. And indeed this would be the natural way to balance any similar physical flow system model, because this is the way to balance the flows in and out of the system.

However, from a point of view of statistical analysis, I believe it would be more fruitful to show a different balance as:

$$\sum e_{i,t} - \sum \tau = \sum w_{i,t}\Omega - \sum w_{i,t}r \qquad \text{or:}$$

$$\sum (e_{i,t} - \tau) = \sum w_{i,t}(\Omega - r) \qquad (7.4e)$$

This gives additive (but flowing) things on the left hand side of the exchange system and multiplicative (flowing) things on the right hand side of the exchange system.

Given that τ, r and Ω are all constants it also reduces a somewhat complex flow system to an exchange system with only two variables, the earnings, $e_{i,t}$, on the left hand side and the wealth, $w_{i,t}$, on the right hand side.

This, I believe, is close to the base model that Sinha was describing; an asymmetric exchange model.

In equation (7.4e) the left hand side additive flows must balance with the right hand side multiplicative flows.

In a normal exchange model both sides of equation (7.4e) would be additive, and indeed identical.

I believe this model, with only two variables and lots of boundary conditions, may be simple enough to be tractable to a traditional statistical mechanical analysis on the lines of Dewar, or indeed Champernowne.

Before moving into further discussion I would first like to follow the maths through a little more. I would like to do two things. Firstly I would like to neglect τ for the moment; we will come back to τ later. Secondly I would like to divide by Ω. That then gives us the following:

$$\sum\left(\frac{e_{i,t}}{\Omega}\right) = \sum w_{i,t}\left(1 - \frac{r}{\Omega}\right) \qquad (7.4f)$$

This brings us back to some old friends. The term (1-r/Ω) gave us our definition of α, the exponent of the powertail, included in equation (4.5q). Equation (7.4f) itself is just a restatement of Bowley's law as defined in equation (1.3x) of this paper. These relations imply that the suggested approach in this section may have promise.

A second observation, which may be completely wrong, is that equation (7.4e) has the feel of a simple differential equation, with wealth on one side, and earnings, the time derivative of wealth, on the other. Instinctively the solution of this would be of exponential form.

Given that the solution of a symmetric exchange is Maxwell-Boltzmann with an exponential tail, then a solution of (7.4e) could reasonably be expected to be a Maxwell-Boltzmann with an exponential-exponential, or a power law tail, as per Reed and Hughes or Baek, Bernhardsson and Minnhagen or others [Reed & Hughes 2002, Baek et al 2011].

An alternative approach is to look at equation (7.4e) from a maximum entropy, statistical mechanical point of view, but now you need to maximise the entropy over two different distributions.

On the left hand side, you have a traditional additive term that should produce a standard Maxwell-Boltzmann distribution of earnings. On the right-hand side you also have a distribution to maximise, however in this case the distribution is multiplicative, and so the ladder of energy levels are proportionately distributed. So the resultant Maxwell-Boltzmann is exponential-exponential, or power law. This seems very close to the original model built by Champernowne, and rediscovered by Levy & Solomon [Simkin & Roychowdhury 2006].

It may be possible to maximise each of these entropies independently, however it seems likely that the distributions on each side will affect each other.

At this point it is worth looking at the left hand side in more detail, as this may answer a quandary discussed back in section 1.1, though it raises as many questions as it answers. In this section it was noted that returns from waged employment appear to follow an offset Maxwell-Boltzmann distribution, or an 'additive GLV distribution'.

Looking at equation (7.4e) the answer to why earnings are distributed as a Maxwell-Boltzmann becomes, in one sense, trivial.

The distribution is a Maxwell-Boltzmann because that is the maximum entropy solution for the distribution of earnings. For a statistical mechanic that is good enough.

Indeed, statistical mechanics would predict a Maxwell-Boltzmann distribution of earnings even when all the individuals had identical skills.

However two questions are raised immediately; why is it offset? And what is the actual mechanism for creating the distribution?

The first question is one for which the answer is not at first obvious. Intuitively, the maximum entropy distribution would extend to zero, because, given a fixed total amount of incomes, this would also allow the maximum values of earnings in the tail to increase, and so give a wider total spread, which would have a higher overall maximum entropy.

However, although the model above attempts to reduce the system to an exchange model, it must be remembered that it is a flow system that is being analysed. I believe that Dewar is absolutely correct that these systems must be modelled by maximising the entropy flow, not just by maximising the entropy.

So, with two distributions, one on each side of the exchange, the simplistic (traditional) solution would be to maximise the joint entropy of the two distributions; that is to multiply the two different partition functions and maximise the single resultant function. However, both distributions are modelling distributions of flow. As well as maximising the entropy embodied in the two distributions, there is a simultaneous need to maximise the entropy embodied in the size of the flows. Hopefully this will be a straightforward trade off between the three (four?) different entropies being enumerated. Intuitively, given this extra contribution to total entropy from the flow, an offset Boltzmann distribution may achieve extra entropy flow to compensate for its narrower spread and the lower entropy in its distribution.

Going back to the concept of dissipative structures and negentropy generators, a narrower Boltzmann distribution for earnings could be seen as a dissipative structure in its own right. This distribution has lower entropy, but is capable of allowing larger entropy flows through the system. Ultimately, if it allowed very high entropy flows the earnings distribution might even collapse into a very low entropy uniform distribution, or, as is often seen in both real world monopolies and many econophysics models, all wealth and income would go to one individual.

With a dissipative structure approach, presumably there is a negentropy flow associated with 'maintaining' the dissipative structure in its low entropy form; Maxwell's demon is continually at work narrowing the spread of the earnings distribution. However if this negentropy flow is smaller than the entropy flow through the system, enabled by the dissipative structure, then the flow system as a whole, including the dissipative structure can be stable and long-lived.

In the example of economics, as long as a factory is making money, it is worth diverting part of the profits to maintain it. If a proposed new factory is predicted to be profitable in the long-term, it is worth borrowing money to build it.

The second question; of the mechanism for creating income distributions, is also problematic.

For the right hand side of equation (7.4e) the mechanism of wealth condensation producing a feed-back loop for increasing wealth via returns on assets discussed in YMTU seems, to me at least, very plausible.

The self-organisation of salaries into a Maxwell-Boltzmann distribution is a harder process to visualise; people do not randomly exchange jobs and salaries with each other.

The first problem is letting go of the fundamental economic belief that people are fairly rewarded for their employment. In fact when employers take on new employees they don't do a detailed analysis of the individual's probable contribution of wealth to the company. They decide if the employee is needed, they look at the market rates for the skills required and they pay the going rate. Certainly overall wage levels are checked carefully against total revenues, and deadwood is chopped back wherever possible. But wages are set externally in the market, not internally by potential wealth creation.

Note also, that in a stable economy, the total sum Σe of earnings available will be fixed, giving the boundary condition necessary for a Maxwell-Boltzmann distribution to develop.

Given that wages are set in the market, a maximum entropy distribution becomes more possible. As long as there is a minimum amount of stochastic churn in the jobs market, with competition and movement up and down a ladder of earnings levels, then creation of a Maxwell-Boltzmann distribution becomes possible.

Moving to a different issue, an element that is missing from all the models of YMTU, is that of unemployment. Wright's models are superior in this regard, and may shed light on this dynamic [Wright 2005 & 2009].

Equation (4.7e), and a Maxwell-Boltzmann distribution, especially an offset one, would seem to imply that all would have jobs and earnings.

I can see two possible causes for persistent mass unemployment. A first explanation is given by reintroducing τ, the compulsory consumption or non-discretionary spending. It is possible that when the values of $e_{i,t}$ at the low end of the distribution becomes less than the value of τ individuals are removed from the distribution altogether.

A second source of persistent unemployment could come from a combination of the maximum entropy flow, dissipative structure model combined with differing actual skill levels. With differing skill levels greater flows of entropy might be achieved by diverting all earnings to highly skilled individuals with no flows to the low skilled. Although the distribution would have lower entropy, total entropy flows might be higher.

Finally, and much more speculatively, I would like to consider what might happen when equation (7.4e) does not balance.

$$\sum (e_{i,t} - \tau) = \sum w_{i,t}(\Omega - r) \qquad (7.4e)$$

I think that equation (7.4e) will balance in many situations of flow systems. Most physical and biological systems will come to a dynamic equilibrium when the flows in and out of the system are equivalent. This will define a pair of distributions and an entropy flow that will have a combined system maximum entropy production.

However for most economic systems the above is not true. Once a market system is installed in a country, the economy starts growing and is characterised by long-term persistent levels of growth. The growth level is so persistent that this can also be characterised as being stable, in that the parameters of the system; gdp growth rate, interest rates, stock-market growth rates, etc, are very stable over decades or even centuries. For newly industrialising economies this is characterised as having high levels of gdp growth up to 10% per annum, with associated high interest rates and stock-market rates. Ω is typically low. For mature economies, gdp growth and interest rates are typically 2-4% and Ω is typically higher.

In these cases Ω can be seen as the external variable. Given this external value of Ω, it could then be possible that there is a set level of gdp growth, interest rates and stock-market returns that gives a maximum entropy production output for the sum of the terms represented by equation (7.4e).

If this was the case then the persistence of endogenous growth would have an explanation.

Even more speculatively, let us reintroduce τ to the discussion. τ will be defined somewhere endogenously within the economic system. It will basically be defined in terms of the proportion of average wage level required to provide basic housing, food, heating, etc. In a developing society it will probably be defined largely by the subsistence wage level needed to provide basic food and shelter. In an advanced economy it will be defined by basic housing rental costs and ultimately the costs of scarce land. This might explain the very similar rates of growth seen in industrialising economies. It could also explain the higher long term growth rates in the US, with its plentiful land compared to the lower rate for the UK, were land has been scarce for centuries.

If τ can be defined endogenously within the system, then Ω should be definable endogenously in terms of τ. People will need to save enough during their working lives to pay for their annual τ during their retirement.

In theory, then the whole system becomes an endogenous equilibrium, with the only real exogenous factor being scarce land prices in advanced economies.

11. Conclusion

It is of course possible that the earnings distributions seen in figures 1.1.4 and 1.1.5 are genuinely exogenous and defined, for example, by earnings ability. If this is the case, then the majority of the discussions above are simply wrong.

It remains however an extraordinary coincidence that these earnings distributions can be fitted by a Maxwell-Boltzmann distribution. It is the belief of the author that economics is providing high quality raw data which may allow the investigation of the detailed statistical mechanics of stable, flowing, 'out of equilibrium systems'.

In the 1970s Paltridge solved the first 'out of equilibrium system' mathematically from a macroscopic point of view. Economics may provide the first 'out of equilibrium' system to be solved exactly from a statistical mechanical point of view. This may mean that the 'offset-Boltzmann' distribution for earnings may be the first 'dissipative structure' that is described exactly in mathematical terms.

11.1 Afterword

This paper is a condensed extract from the full paper 'Why Money Trickles Up'. The full paper applies the same basic model to explain the power tail seen in company size distributions, it also provides models for booms and busts in commodity prices and macroeconomic business cycles. The full paper explains the Bowley ratio; the ratio of returns to labour and capital. The full paper also contains extensive background material on chaos, statistical mechanics, entropy and heterodox economics and finance. The abstract and paper structure for the full paper are given below in section 11.2.

The paper YMTU was researched and written in a little over a year, without financial support or academic supervision.

Foolishly, I have gone against a basic conclusion of this paper, and spent a significant portion of my own capital in producing it.

If you have found the paper of interest or value, any donation to defray the costs of writing it, no matter how small, would be gratefully received.

Those who wish to make a donation can do so by clicking on the Paypal link below:

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11.2 Abstract & Structure of full paper 'Why Money Trickles Up'

Abstract

This paper combines ideas from classical economics and modern finance with Lotka-Volterra models, and also the general Lotka-Volterra models of Levy & Solomon to provide straightforward explanations of a number of economic phenomena.

Using a simple and realistic economic formulation, the distributions of both wealth and income are fully explained. Both the power tail and the log-normal like body are fully captured. It is of note that the full distribution, including the power law tail, is created via the use of absolutely identical agents.

It is further demonstrated that a simple scheme of compulsory saving could eliminate poverty at little cost to the taxpayer. Such a scheme is discussed in detail and shown to be practical.

Using similar simple techniques, a second model of corporate earnings is constructed that produces a power law distribution of company size by capitalisation.

A third model is produced to model the prices of commodities such as copper. Including a delay to capital installation; normal for capital intensive industries, produces the typical cycle of short-term spikes and collapses seen in commodity prices.

The fourth model combines ideas from the first three models to produce a simple Lotka-Volterra macroeconomic model. This basic model generates endogenous boom and bust business cycles of the sort described by Minsky and Austrian economists.

From this model an exact formula for the Bowley ratio; the ratio of returns to labour to total returns, is derived. This formula is also derived trivially algebraically.

This derivation is extended to a model including debt, and it suggests that excessive debt can be economically dangerous and also directly increases income inequality.

Other models are proposed with financial and non-financial sectors and also two economies trading with each other. There is a brief discussion of the role of the state and monetary systems in such economies.

The second part of the paper discusses the various background theoretical ideas on which the models are built.

This includes a discussion of the mathematics of chaotic systems, statistical mechanical systems, and systems in a dynamic equilibrium of maximum entropy production.

There is discussion of the concept of intrinsic value, and why it holds despite the apparent substantial changes of prices in real life economies. In particular there are discussions of the roles of liquidity and parallels in the fields of market-microstructure and post-Keynesian pricing theory.

Structure of the Paper

Part A of this paper discusses a number of economic models in detail, Part A.I discusses a number of straightforward models giving results that easily accord with the real world and also with the models of Ian Wright. Part A.II discusses models that are more speculative.

Part B discusses the background mathematics, physics and economics underlying the models in Part A. The mathematics and physics is discussed in Part B.I, the economics in part B.II, the conclusions are in part B.III. Finally, Part C gives appendices.

Within Part A; section 1 discusses income and wealth distributions; section 1.1 gives a brief review of empirical information known about wealth and income distributions while section 1.2 gives background information on the Lotka-Volterra and General Lotka-Volterra models. Sections 1.3 to 1.5 gives details of the models, their outputs and a discussion of these outputs.

Section 1.6 discusses the effects that changing the ratio of waged income to earnings from capital has on wealth and income distributions.

Sections 1.7 and 1.8 discuss effective, low-cost options for modifying wealth and income distributions and so eliminating poverty.

Finally, section 1.9 looks at some unexplained but potentially important issues within wealth and income distribution.

Sections 2.1 to 2.4 go through the background, creation and discussion of a model that creates power law distributions in company sizes.

Sections 3.1 to 3.4 use ideas from section 2, and also the consequences of the delays inherent in installing physical capital, to generate the cyclical spiking behaviour typical of commodity prices.

Sections 4.1 to 4.4 combine the ideas from sections 1, 2 and 3 to provide a basic macroeconomic model of a full, isolated economy. It is demonstrated that even a very basic model can endogenously generate cyclical boom and bust business cycles of the sort described by Minsky and Austrian economists.

In section 4.5 it is demonstrated that an exact formulation for the Bowley ratio; the ratio of returns to labour to total returns, can easily be derived from the basic macroeconomic model above, or indeed from first principles in a few lines of basic algebra.

In section 4.6 and 4.7 the above modelling is extended into an economy with debt. From this a more complex, though still simple, formulation for the Bowley ratio is derived. This formulation suggests that excessive debt can be economically dangerous and also directly increases income inequality. The more general consequences of the Bowley ratio for society are discussed in more depth in section 4.8.

In section 4.9 two macroeconomic models are arranged in tandem to discuss an isolated economy with a financial sector in addition to an ordinary non-financial sector. In section 4.10 two macroeconomic models are discussed in parallel as a model of two national economies trading with each other.

To conclude Part A, section 4.11 introduces the role of the state and monetary economics, while section 4.12 briefly reviews the salient outcomes of the modelling for social equity.

In Part B, section 6.1 discusses the differences between static and dynamic systems, while section 6.2 looks at the chaotic mathematics of differential equation systems. Examples of how this knowledge could be applied to housing markets is discussed in section 6.3, while applications to share markets are discussed in section 6.4. A general overview of the control of chaotic systems is given in section 6.5.

Section 7.1 discusses the theory; 'statistical mechanics', which is necessary for applying to situations with many independent bodies; while section 7.2 discusses how this leads to the concept of entropy.

Section 7.3 discusses how systems normally considered to be out of equilibrium can in fact be considered to be in a dynamic equilibrium that is characterised as being in a state of maximum entropy production. Section 7.4 discusses possible ways that the statistical mechanics of maximum entropy production systems might be tackled.

Moving back to economics; in section 8.1 it is discussed how an intrinsic measure of value can be related to the entropy discussed in section 7 via the concept of 'humanly useful negentropy'.

Section 8.2 discusses the many serious criticisms of a concept of intrinsic value in general, with a discussion of the role of liquidity in particular.

Section 9.1 looks at theories of supply and pricing, the non-existence of diminishing returns in production, and the similarities between the market-microstructure analysis and post-Keynesian pricing theory. Section 9.3 looks for, and fails to find, sources of scarcity, while section 9.4 discusses the characteristics of demand.

In section 10 both the theory and modelling is reviewed and arranged together as a coherent whole, this is followed by brief conclusions in section 11.

Sections 12 to 16 are appendices in Part C.

Section 12 gives a history of the gestation of this paper and an opportunity to thank those that have assisted in its formation.

Section 13 gives a reading list for those interested in learning more about the background maths and economics in the paper.

Section 14 gives details of the Matlab and Excel programmes used to generate the models in Part A of the paper.

Sections 15 and 16 give the references and figures respectively.

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16. Figures

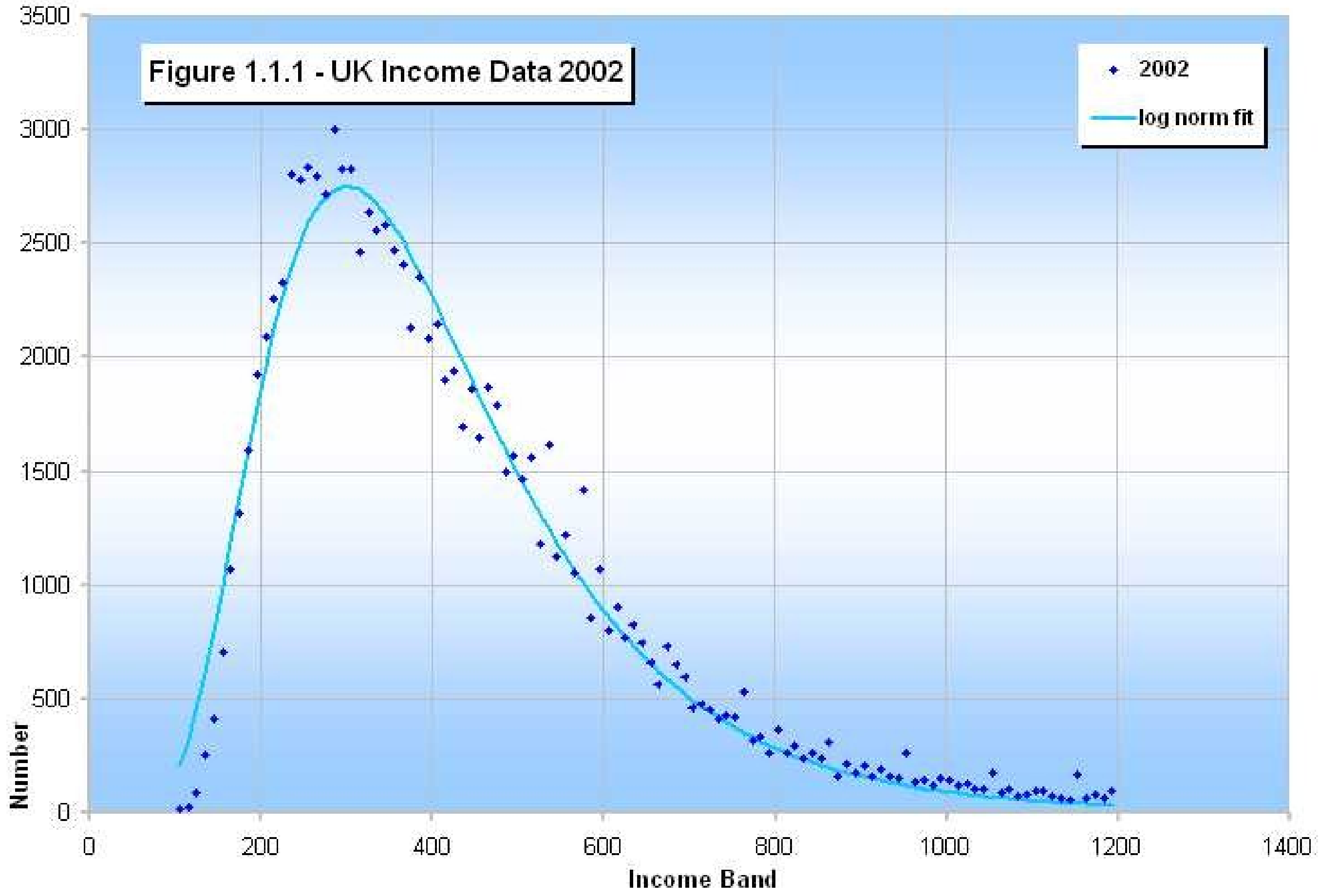


Figure 1.1.1 - UK Income Data 2002
2002
log norm fit
Number
Income Band
3500
3000
2500
2000
1500
1000
500
0
0
200
400
600
800
1000
1200
1400

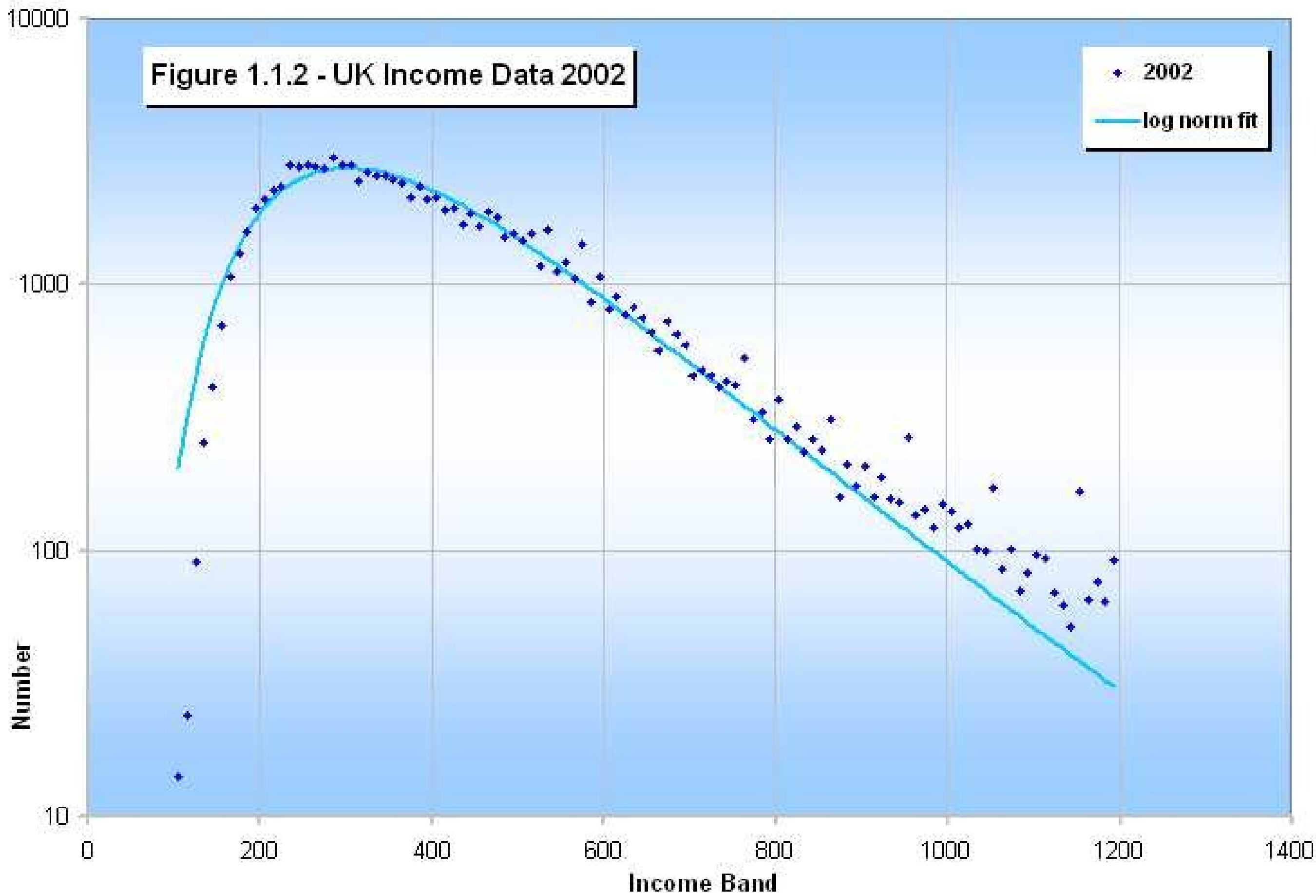


Figure 1.1.2 - UK Income Data 2002
2002
log norm fit
Number
Income Band
10000
1000
100
10
0
200
400
600
800
1000
1200
1400

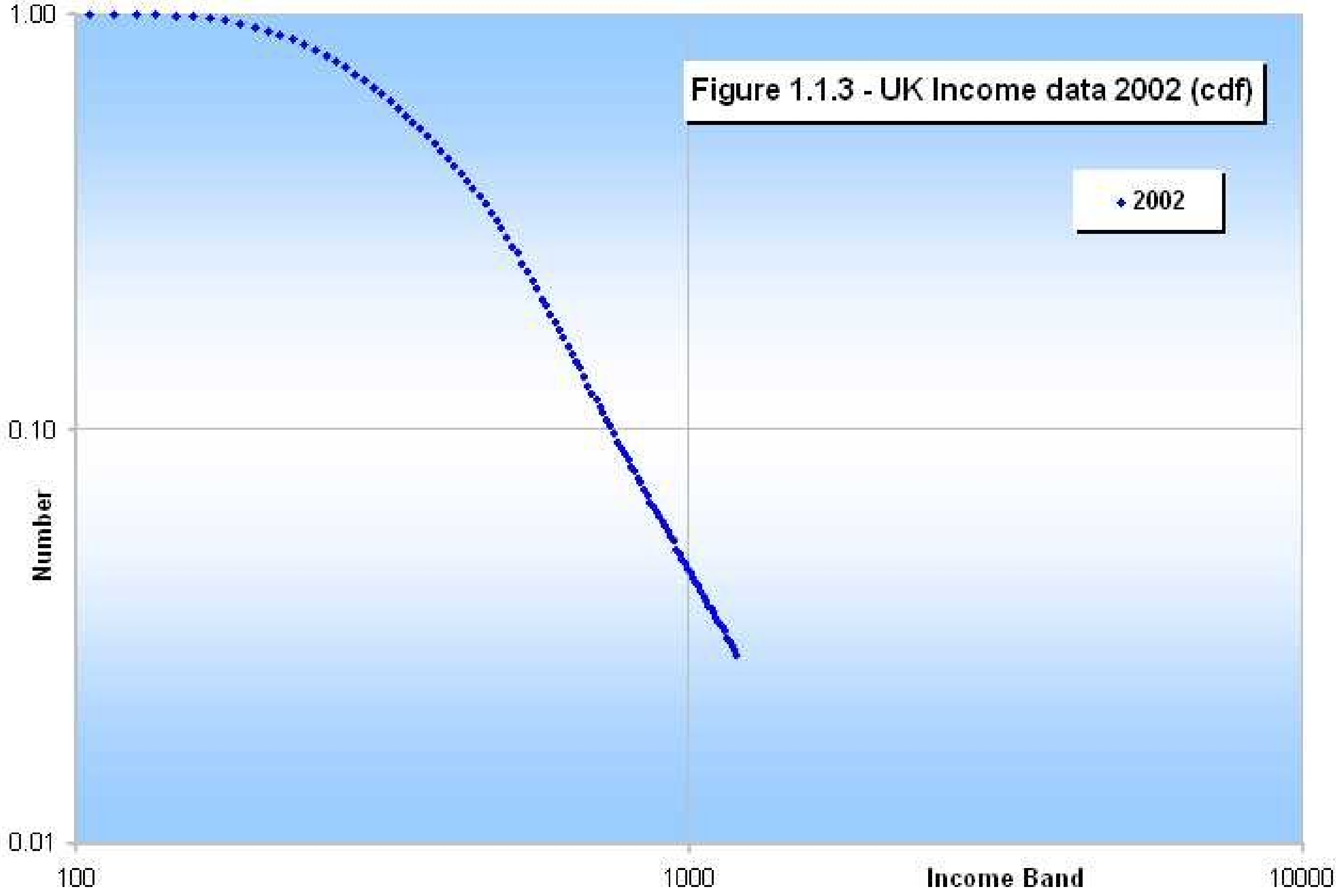


Figure 1.1.3 - UK Income data 2002 (cdf)
2002
1.00
0.10
0.01
Number
100
1000
10000
Income Band

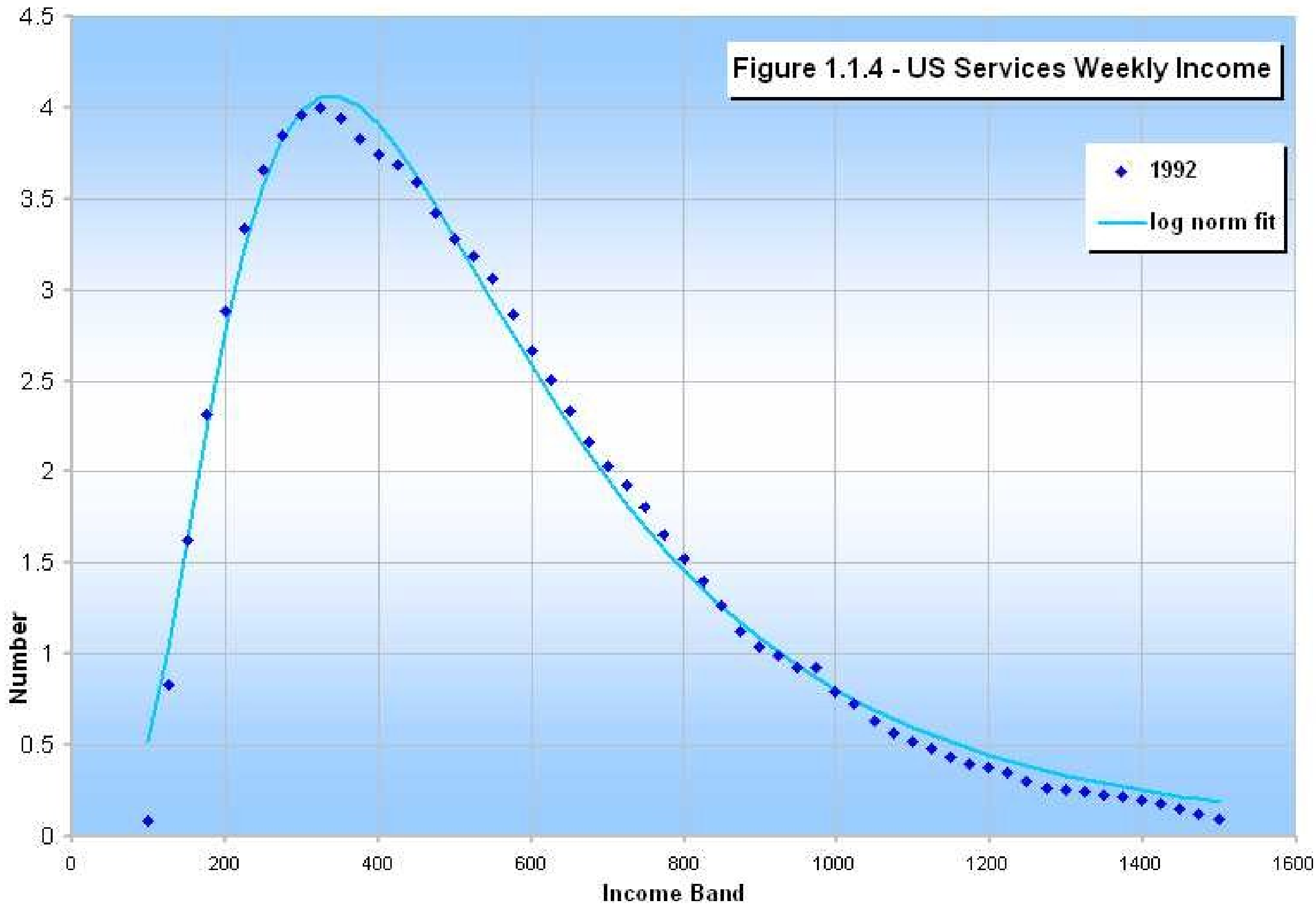


Figure 1.1.4 - US Services Weekly Income
1992
log norm fit
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
Number
0
200
400
600
800
1000
1200
1400
1600
Income Band

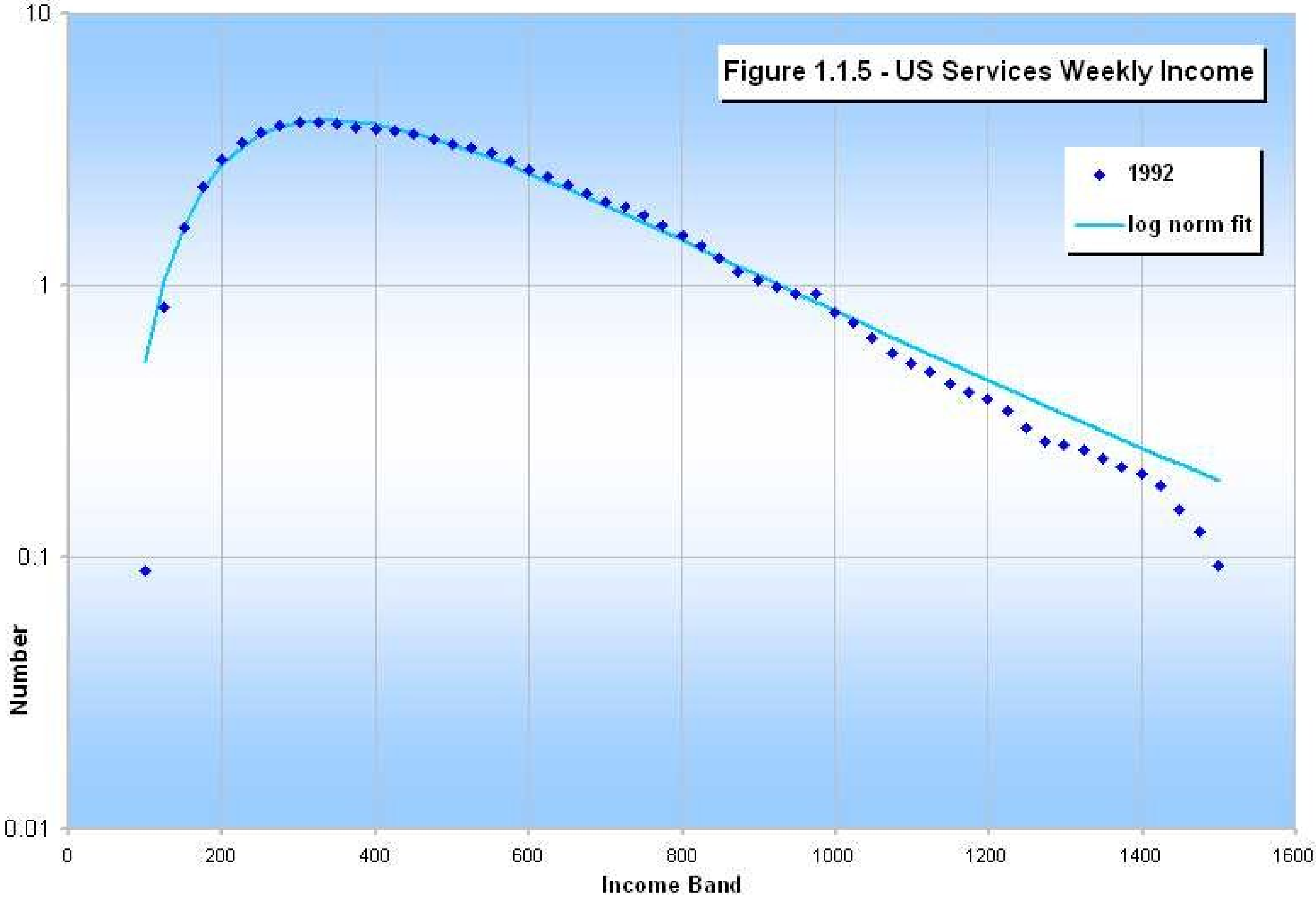


Figure 1.1.5 - US Services Weekly Income
1992
log norm fit
Number
Income Band
10
1
0.1
0.01
0
200
400
600
800
1000
1200
1400
1600

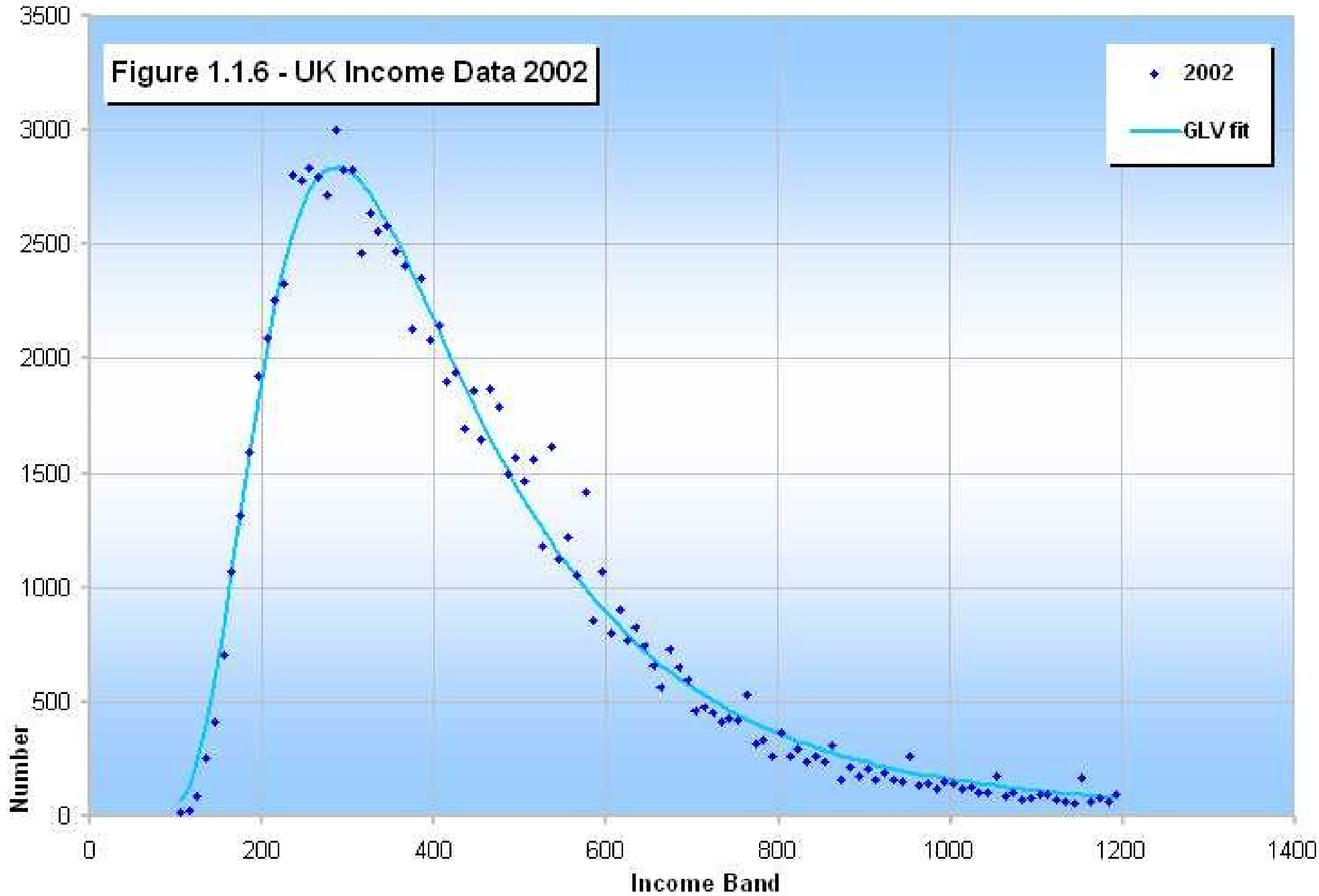


Figure 1.1.6 - UK Income Data 2002
2002
GLV fit
Number
Income Band
3500
3000
2500
2000
1500
1000
500
0
0
200
400
600
800
1000
1200
1400

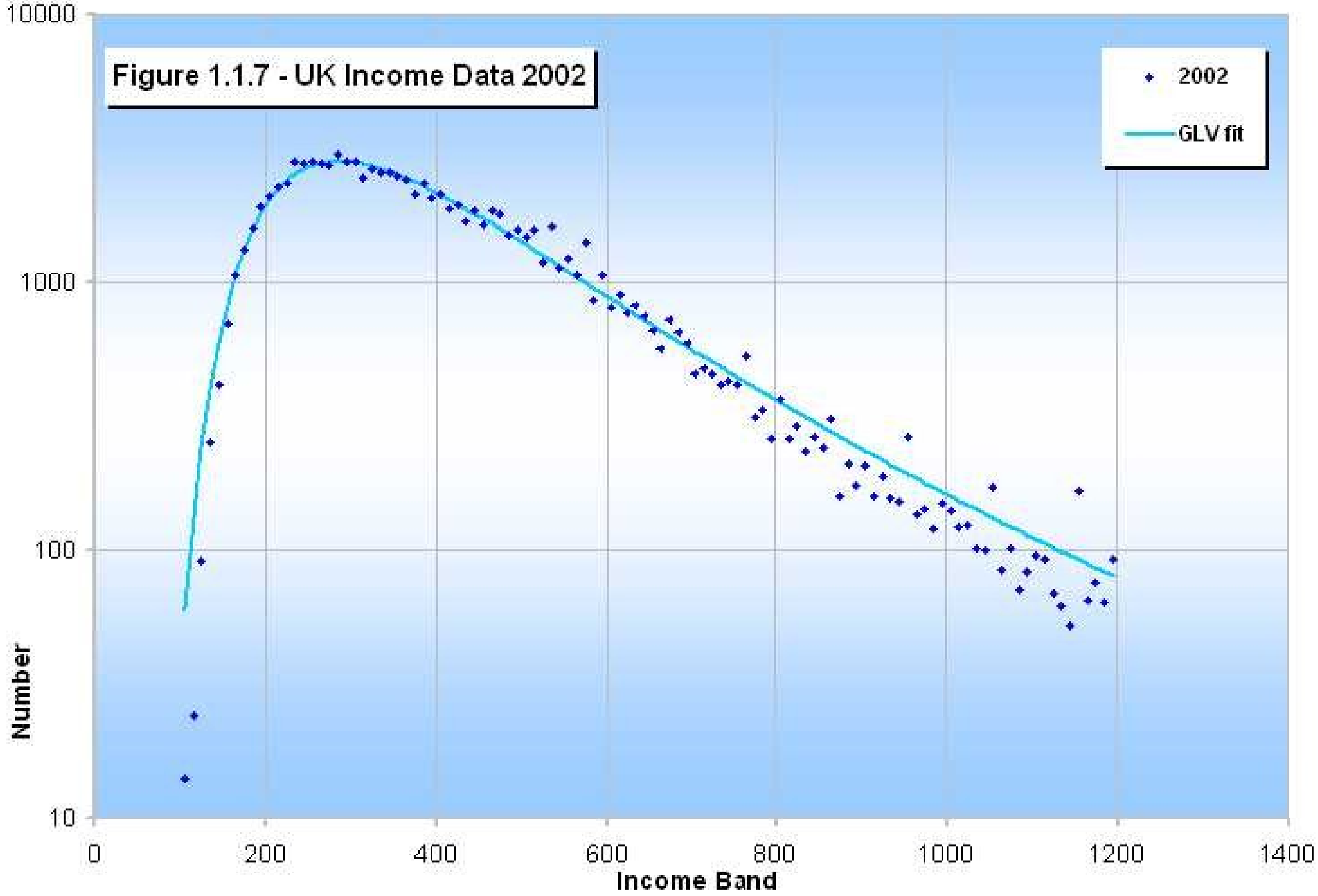
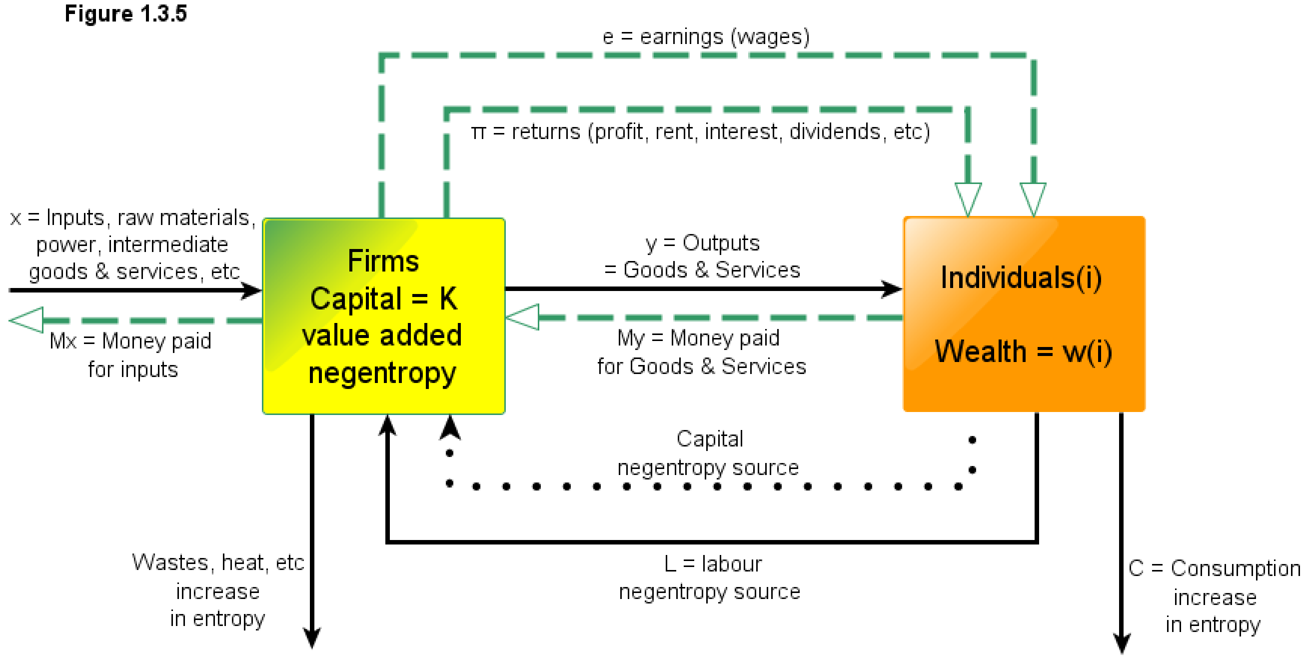
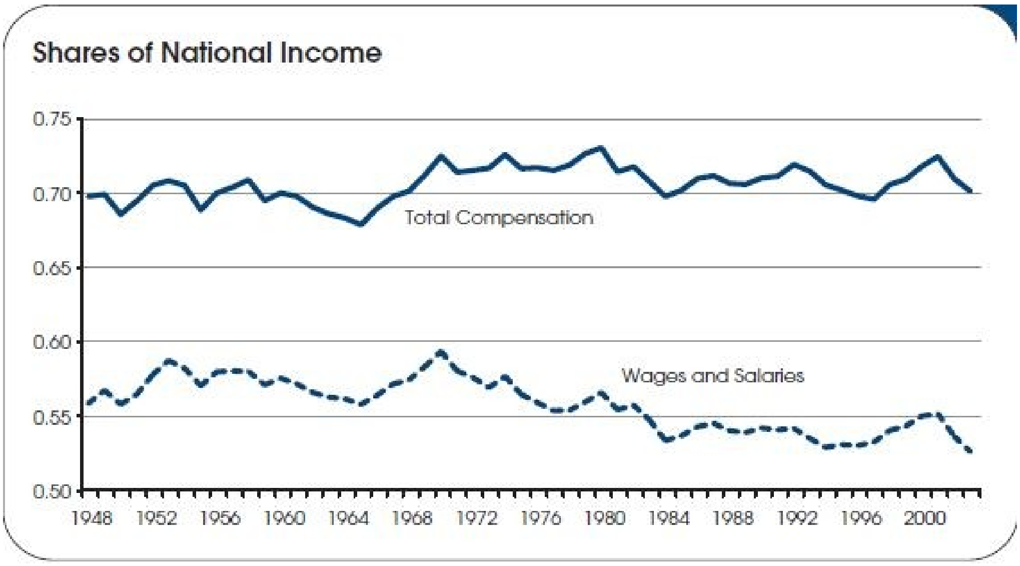


Figure 1.1.7 - UK Income Data 2002
2002
GLV fit
10000
1000
100
10
Number
0
200
400
600
800
1000
1200
1400
Income Band

Figure 1.3.5


e = earnings (wages)
π = returns (profit, rent, interest, dividends, etc)
x = Inputs, raw materials, power, intermediate goods & services, etc
Mx = Money paid for inputs
Firms
Capital = K
value added
negentropy
y = Outputs
= Goods & Services
My = Money paid
for Goods & Services
Individuals(i)
Wealth = w(i)
Capital
negentropy source
L = labour
negentropy source
Wastes, heat, etc
increase
in entropy
C = Consumption
increase
in entropy


Shares of National Income
0.75
0.70
0.65
0.60
0.55
0.50
Total Compensation
Wages and Salaries
1948
1952
1956
1960
1964
1968
1972
1976
1980
1984
1988
1992
1996
2000
Figure 1.3.8

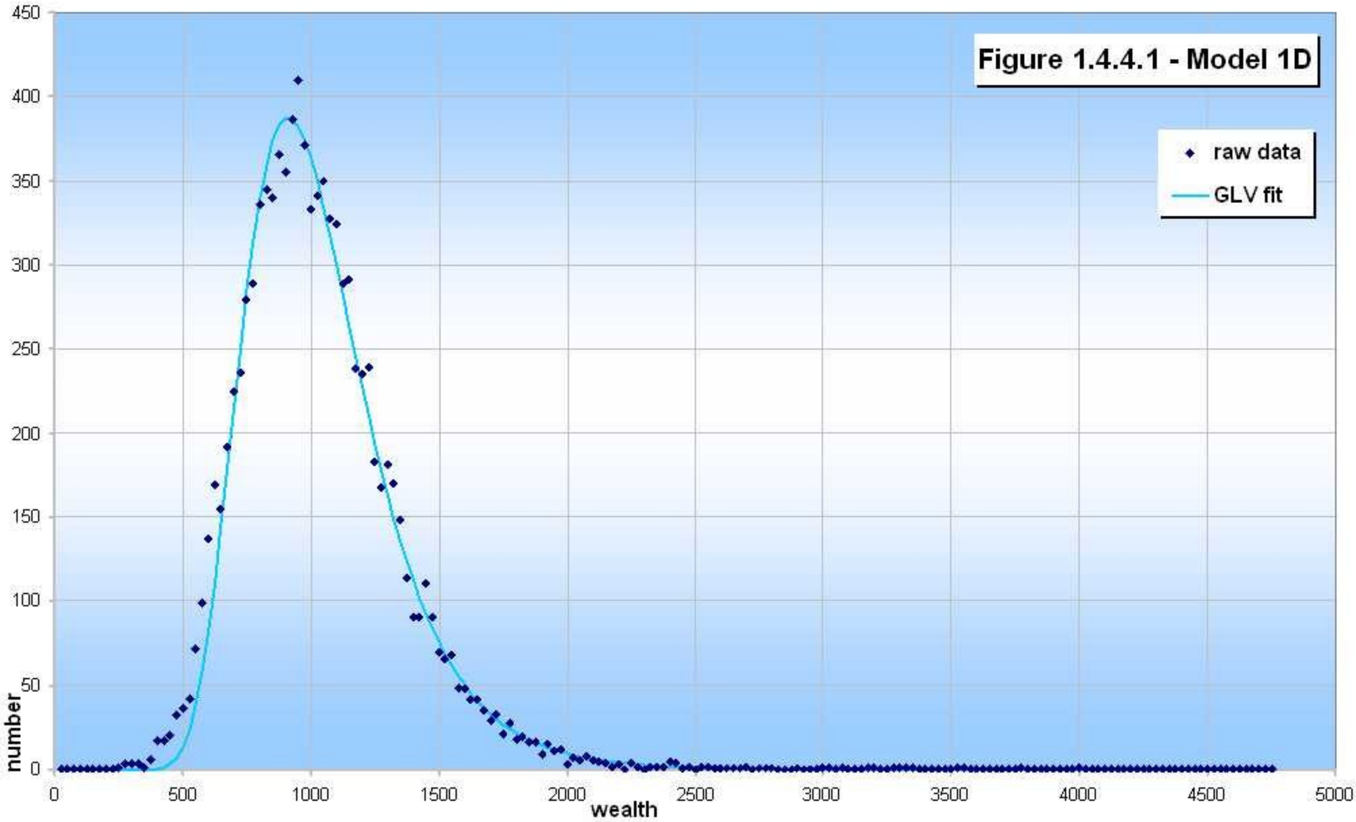


Figure 1.4.4.1 - Model 1D
raw data
GLV fit
number
wealth
0
50
100
150
200
250
300
350
400
450
500
1000
1500
2000
2500
3000
3500
4000
4500
5000

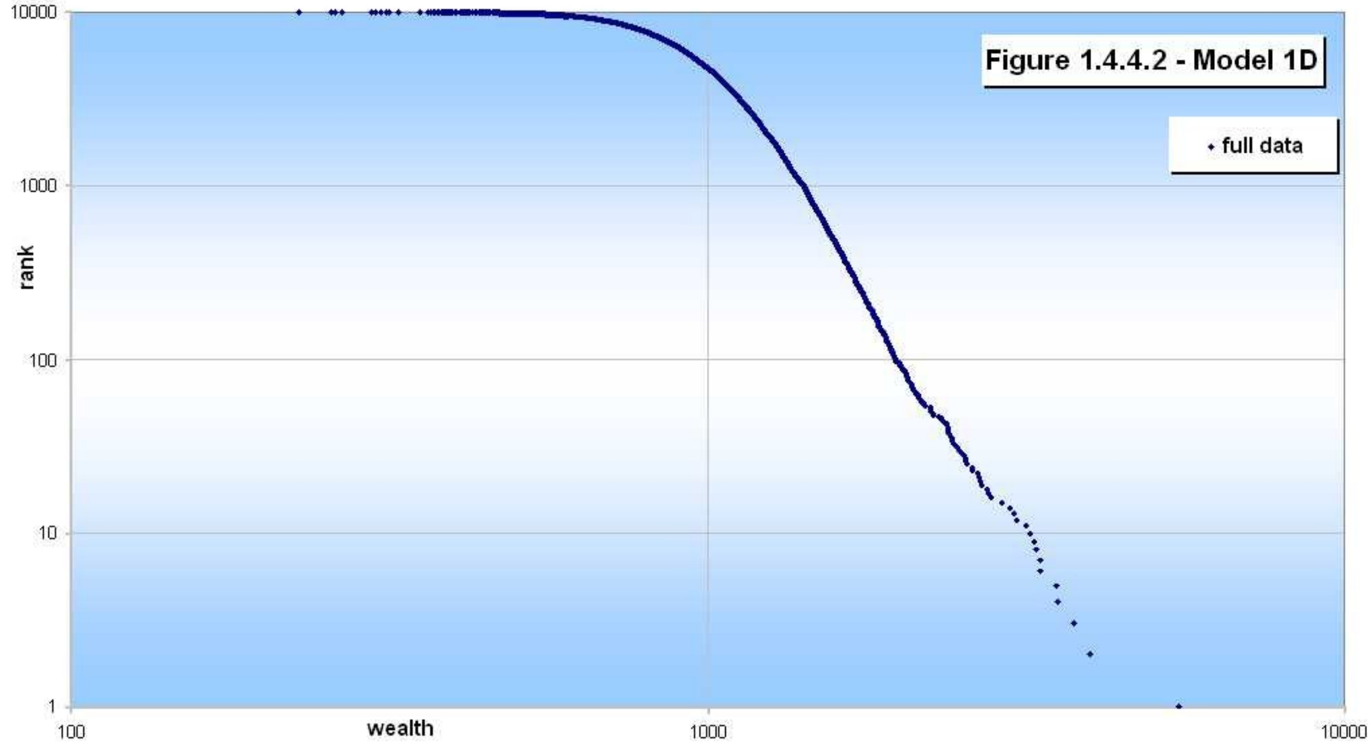


Figure 1.4.4.2 - Model 1D
full data
rank
wealth
10000
1000
100
10
1
100
1000
10000

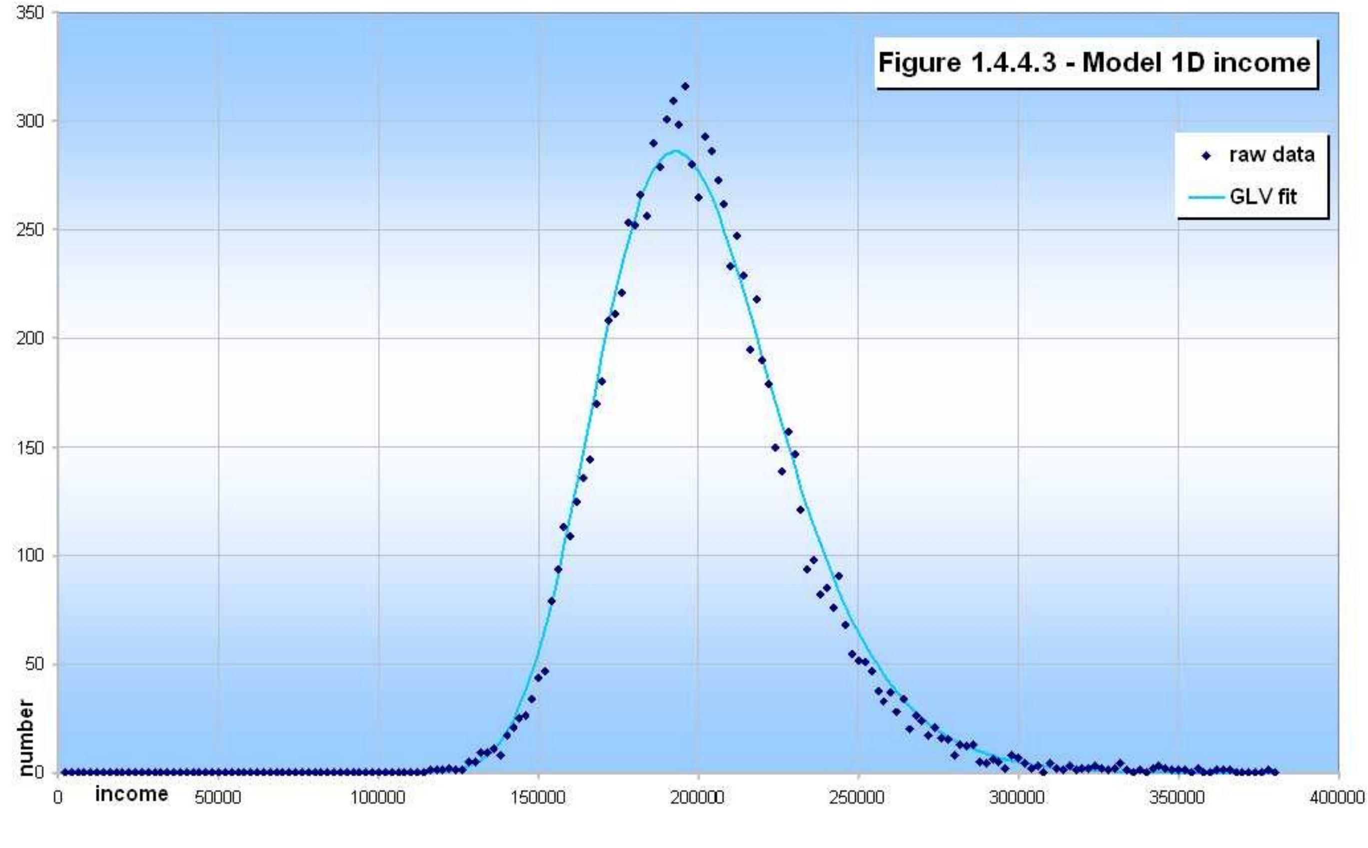


Figure 1.4.4.3 - Model 1D income
raw data
GLV fit
number
income
350
300
250
200
150
100
50
0
0
50000
100000
150000
200000
250000
300000
350000
400000

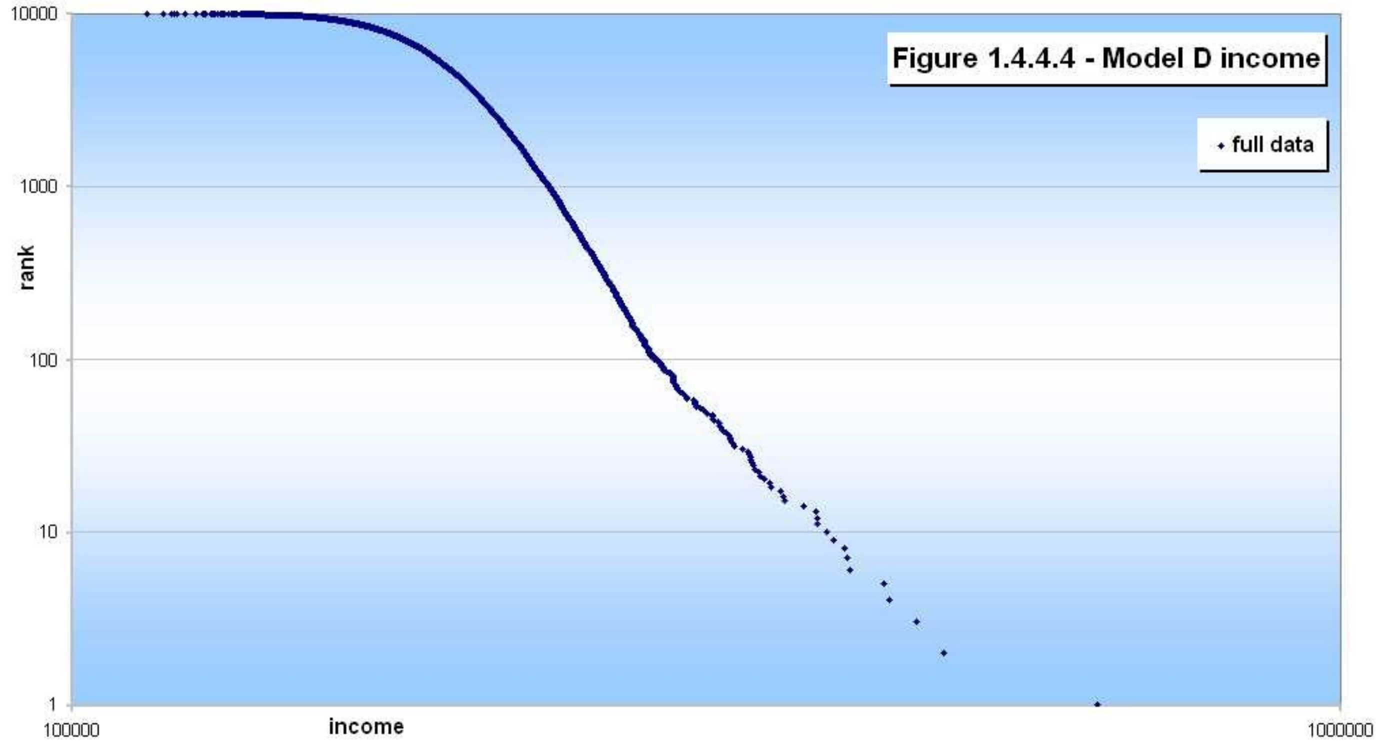


Figure 1.4.4.4 - Model D income
full data
rank
income
10000
1000
100
10
1
100000
1000000

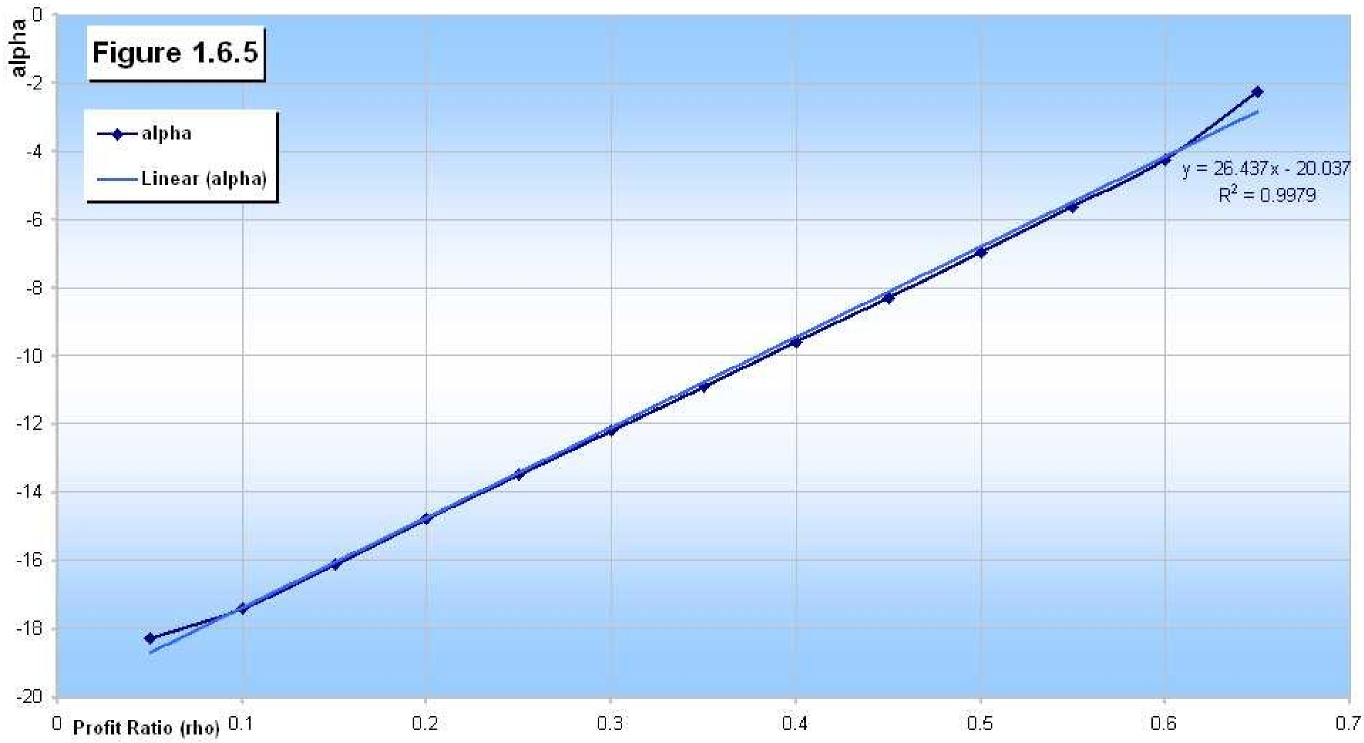


Figure 1.6.5
alpha
Linear (alpha)
y = 26.437x - 20.037
R² = 0.9979
Profit Ratio (rho)
0
-2
-4
-6
-8
-10
-12
-14
-16
-18
-20
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7